A two-dimensional MHD free convection heat and mass transfer flow of viscous, incompressible and electrically conducting fluid past a vertical flat plate embedded in porous medium in the presence of hall current under the influence of uniform magnetic field applied normal to the flow is studied analytically. In this research work, we make the governing equations dimensionless by usual non-dimensional variables and we obtained a set of ordinary differential equations. Then these obtained ordinary differential equations are solved analytically by using perturbation technique. The expressions for velocity field, temperature distribution, concentration field, skin friction, the rate of heat transfer and the rate of mass transfer are derived. Finally the results are discussed in detailed with the help of graphs and tables to observe the effect of different parameters like Magnetic parameter (M), radiation parameter (F), Grashof number (Gr), modified Grashof number (Gm), Prandtl number (Pr), permeability parameter (k), Eckert number (Ec) and the chemical reaction parameter (Kc).

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Dedicated to

...my parents and grandparents

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Abstract

A two-dimensional MHD free convection heat and mass transfer flow of viscous, incompressible and electrically conducting fluid past a vertical flat plate embedded in porous medium in the presence of hall current under the influence of uniform magnetic field applied normal to the flow is studied analytically. In this research work, we make the governing equations dimensionless by usual non-dimensional variables and we obtained a set of ordinary differential equations. Then these obtained ordinary differential equations are solved analytically by using perturbation technique. The expressions for velocity field, temperature distribution, concentration field, skin friction, the rate of heat transfer and the rate of mass transfer are derived. Finally the results are discussed in detailed with the help of graphs and tables to observe the effect of different parameters like Magnetic parameter (M), radiation parameter (F), Grashof number (Gr), modified Grashof number (Gm), Prandtl number (Pr), permeability parameter (k), Eckert number (Ec) and the chemical reaction parameter (Kc).

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Nomenclature

x	Coordinate axis along to the plate
y	Coordinate axis normal to the plate
y	Dimensionless coordinate axis normal to the plate
u	Velocity components along x-direction
u	Dimensionless velocity
v	Velocity components along y-direction
B_0	Uniform magnetic field
C	Species concentration of the fluid
C_w	Species concentration of the fluid at the plate
C	Species concentration of the fluid far away from the plate
C_p	Specific heat at constant pressure
T	Temperature of the fluid near the plate
T_w	Temperature of the fluid at the plate
T	Temperature of the fluid far away from the plate
g	Acceleration due to gravity
D	Chemical molecular diffusivity
m	Hall parameter
Gr	Grashof number
Gm	Modified Grashof number
Sc	Schmidt number
Nu	Nusselt number
Sh	Sherwood number
Pr	Prandtl number

Ec	Eckert number
k	Permeability parameter
F	Radiation parameter
K_c	Chemical reaction parameter
K_p	Porous medium factor
K	Dimensionless porous medium factor
M	Dimensionless magnetic field parameter
q_r	Radiative heat flux in the y direction
v_0	Uniform velocity

Greek symbols

β	Coefficient of thermal expansion
$\bar{\beta}$	Coefficient of thermal expansion with concentration
	Coefficient of viscosity
υ	Kinematic viscosity
	Density of the fluid
	Electric conductivity
	Dimensionless skin friction
	Dimensionless concentration
κ	Thermal conductivity

Subscripts

w	Conditions on the wall
	Free stream conditions

Superscript

	Differentiation with respect to y

Chapter 1

Introduction

Magneto-hydrodynamics (MHD) is the science which deals with the motion of a highly conduction fluid in the presence of magnetic field. The motion of conducting fluid across the magnetic field on these currents which change the magnetic field and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid. The field of Magneto-hydrodynamic (MHD) was initiated by Hannes Alfven (1955), for which he received the Nobel Prize in physics in 1970. It is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Probably the largest advances towards an understanding of such phenomena come from the field of astrophysics. It has long been suspected that most of the matter in the universe is in the plasma or highly ionized gaseous state and much of the basic knowledge in the area of electromagnetic fluid dynamics evolved from these studies. As a branch of plasma physics the field of Magneto-hydrodynamics consists of the study of a continuous, electrically conducting fluid under the influence of electromagnetic fields. Originally, MHD included only the study of strictly incompressible fluid but today the terminology is applied to studies of partially ionized gases as well. Other names have been suggested such as magneto fluid-mechanics or Magneto hydrodynamics but original nomenclature has persisted. The essential requirement for problems to be analyzed under the laws of MHD is that the continuum approach be applicable. Many natural phenomena and engineering problems are worth being subjected to an MHD analysis. Furthermore, Magneto-hydrodynamic has attracted the attention of a large number of scholars due to its diverse applications. In engineering it finds its application in MHD pumps, MHD bearings etc. The principal MHD effects were first demonstrated in the experiments of Faraday and Ritchie.

Faraday carried out experiments with the flow of mercury in glass tubes placed between poles of a magnet and discovered that a voltage was induced across the tube due to the motion of the mercury across the magnetic fields, perpendicular to the direction of flow and to the magnetic field. Soundalgekar and Takhar [1] first, studied the effect of radiation on the natural convection flow of a gas past a semi-infinite plate using the Cogly-Vincentine-Gilles equilibrium model. For the same gas Takhar *et al.* [2] investigated the effects of radiation on the MHD free convection flow past a semi-infinite vertical plate. Later Hossain *et al.* [3] studied the effect of radiation on free convection from a porous vertical plate. Recently the thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion has been studies by Muthucummaraswamy and Vislakshi [4].

The combined effect of viscous dissipation, Joule heating, transpiration, heat source, thermal diffusion and Hall current on the hydro-Magnetic free convection and mass transfer flow of an electrically, viscous, homogeneous, incompressible fluid past an infinite vertical porous plate are discussed by Singh *et al.* [5]. Singh [6] has also studied the effects of mass transfer on MHD free convection flow of a viscous fluid through a vertical channel walls.

The problem of convection heat transfer in a porous media is a topic of rapidly growing interest due to its applications to geophysics, geothermal reservoirs, thermal insulation engineering, exploration of petroleum and gas fields, water movements in geothermal reservoirs etc. Soundalgekar [7] obtained approximate solutions for the two-dimensional flow of an incompressible, viscous fluid past an infinite vertical porous plate with constant suction normal to the plate. Murali *et al.* [8] examined the thermal radiation effect on unsteady magneto hydrodynamic flow past a vertical porous plate with variable suction. Damala *et al.* [9] make a study on the effect of the steady two-dimensional free convection heat and mass transfer flow electrically conducting and chemically reacting fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented. The effects of chemical reaction and radiation absorption have been discussed on unsteady MHD free convection heat and mass transfer flow on a viscous, incompressible, electrically conducting fluid past a semi-infinite inclined porous plate, moving with a uniform velocity are discussed in Sudersan *et al.* [10]. An analytical solution for unsteady free convection in porous media has been studied by Magyari *et al.* [11].

Muthucumaraswamy and Janakiraman [12] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. An exact solution to one dimensional unsteady natural convection flow past an infinite vertical accelerated plate, immersed in a viscous thermally stratified fluid is investigated by Rudra and Bhaben [13]. Tasawar *et al.* [14] investigated the influence of radiation on magneto hydrodynamic (MHD) and mass transfer flow over a porous stretching sheet. Muthucummaraswamy *et al.* [15] presented an exact analysis of rotation effects on unsteady flow of an incompressible and electrically conducting fluid past uniformly accelerated infinite vertical plate, under the action of electrically conducting fluid past a uniformly accelerated infinite vertical plate, under the action of transversely applied magnetic field. An analytical study is performed to examine the effects of temperature dependent heat source on the unsteady free convection and mass transfer flow of an elasto- viscous fluid past an exponentially accelerated infinite vertical plate in the presence of magnetic field through porous medium by Rajesh [16]. Suneetha *et al.* [17] investigated thermal radiation effects on MHD flow past an impulsively started vertical plate in the presence of heat source/ sink by taking into account the heat due to viscous dissipation. The governing boundary layer equations of the flow field are solved by an implicit finite difference method of Crank Nicholson type. Effects on boundary layer flow and heat transfer of a fluid with variable viscosity along a symmetric wedge is presented here by Mukhopadhyay [18]. Rajput and Surendra [19] studied the MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Theoretical solution of unsteady flow past an uniformly accelerated infinite vertical plate has been presented Muthucumaraswamy *et al.* [20] in the presence of variable temperature and uniform mass diffusion. Rajesh [21] discussed the effect of a uniform transverse magnetic field on the free convection and mass- transform flow of an electrically- conducting fluid past an exponentially accelerated infinite vertical plate through a porous medium. Sing *et al.* [22] studied the two dimensional free convection and mass transfer flow of an incompressible viscous and a continuously moving infinite vertical porous plate in the presence of heat source, thermal diffusion, large suction and under the influence of uniform magnetic field applied normal to the flow is studied and perturbation technique is used to solve the governing equation Mehmood and Ali [23] studied the effect of the wall slip on velocity field. By Rajesh and Vijaya [24] an analytical study is performed to study the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field.

In my present work, I study about the effects of thermal radiation and chemical reaction on mass transfer on unsteady free convection flow past an exponentially accelerated infinite vertical plate through porous medium in the presence of magnetic hall current. The dimensionless governing equations are reduced to a set of ordinary differential equation. Then I solve these equations with the help of transformed boundary conditions.

Chapter 2

Some Available Information

2.1 Magneto hydrodynamics (MHD)

Magneto hydrodynamics (MHD) is a branch of magneto fluid dynamics *i.e.* continuum mechanics, which deals with the flow of electrically conducting fluids in electric and magnetic fields. The largest advance towards an understanding of such phenomena probably comes from the field of astrophysics. It has long been suspected that most of the matter in the universe is in the plasma or high ionized gaseous state and much of the basic knowledge in the area of electromagnetic field dynamics evolved from these studies.

The fluid of Magneto hydrodynamics consists of the study of a continuous, electrically conducting fluid under the influence of electromagnetic fields, as a branch of plasma physics. Originally, MHD included only the study of strictly incompressible fluid but today the terminology is applied to studies of partially ionized gases as well as the other names have been suggested, such as magneto-fluid-mechanics or magneto-aero-dynamics, but original nomenclature has persisted. The essential requirement for problems to be analyzed under the law of MHD is that the continuum approach be applicable.

There are many natural phenomena and engineering problems are susceptible to MHD analysis. It is useful in astrophysics because much of the universe is filled with widely spaced charged particles and permeated by magnetic fields and so the continuum assumption becomes applicable. Geophysicists encounter MHD phenomena in the interactions of conducting fluid and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, pumps and flow meters, in space vehicle propulsion, control and reentry problem, in designing communications and radar system, in creating novel power generating systems and in developing confinement schemes for controlled fusion.

The most important application of MHD is in the generation of electrical power with the flow of an electrically conducting fluid through a transverse magnetic field. Recently, experiments with ionized gases have been performed with the hope of producing power on a large scale in stationary plants with large magnetic fields. Cryogenic and superconducting magnets are required to produce these very large magnetic fields. Generation of MHD power on a smaller scale is of interest for space applications.

Generally it is known that, several intermediate transformations are necessary to convert the heat energy into electricity. Each of these steps means a loss of energy. This naturally limits the overall efficiency, reliability and compactness of the conversion process. Methods for direct conversion to energy are now increasingly receiving attention. Of these, the fuel cell converts the chemical energy of fuel directly into electrical energy, fusion energy utilizes the energy released when two hydrogen nuclei fuse into a heavier one, and thermo electrical power generation uses a thermocouple. Magneto hydrodynamics power generation is another important new process that is receiving worldwide attention.

In the experiment of Farady (1832), the principal MHD effects were first demonstrated. He discovered that a voltage was induced across the tube due to the motion of the mercury across the magnetic fields, perpendicular to the direction of flow and to the magnetic field by the experiment of the flow of mercury in glass tubes placed between poles of a magnet. Farady (1832), also suggested that electrical power could be generated in a load circuit by the interaction of a following conducting fluid and a magnetic field.

Alfven (1942) discovered MHD waves in the sun. These waves are produced by disturbances which propagate simultaneously in the conducting fluid and the magnetic field. The analogy that explains the generation of an Alfven wave is that of a harp string plucked while submerged in a fluid. The string provides elastic force and the fluid provides inertia force and they combine to propagate a perturbing wave through the fluid and string.

In summary, MHD phenomena result from the mutual effect of a magnetic field and conducting fluid flowing across it. Thus an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluid’s motion. The current also generates its own magnetic field which distorts the original magnetic field. An opposing or pumping force on the fluid can be produced by applying an electric field perpendicularly to the magnetic field. Disturbance in either the magnetic field or the fluid can propagate in both to produce MHD waves as well as upstream and downstream wave phenomena. The science of magneto hydrodynamics is the detailed study of these phenomena, which occur in nature and are produced in engineering devices.

2.2 Flow

Matter exhibits deformation under the action of forces. It is flow if the deformation continuously increases without limit under the action of forces, however small.

2.2.1 Some Important Types of Flow

Steady and Unsteady Flow

A flow in which properties and conditions (say, P) associated with the motion of the fluid are independent of the time so that the flow pattern remains unchanged with the time is said to be steady. Mathematically we may write $\frac{\partial P}{\partial t} = 0.$ Here P may be velocity, density, pressure, temperature etc.

On the other hand a flow in which properties and conditions associated with the motion of the fluid depend on time so that flow pattern varies with time is said to be unsteady

In unsteady flow $\frac{\partial P}{\partial t} \neq 0.$

Uniform and Non-uniform Flow

A flow, in which the fluid particle possesses equal velocities at each section of the channel or pipe is called uniform flow. On the other hand a flow in which the fluid particles possess different velocities at each section of the channel or pipe is called non-uniform flow. These terms are usually used in connection with flow in channels.

2.3 Fluid

Fluid meant a substance that flows. It is an aggregate of particles which yields to the slightest effort made to separate from each other, if it be continued long enough.

2.3.1 Some Important Types of Fluid

Compressible and Incompressible Fluid

A compressible fluid is one in which the fluid density changes when it is subjected to high pressure-gradients. For gasses, changes in density are accompanied by changes in temperature. A fluid is said to be incompressible if it requires a large variation in pressure to produce some appreciable variation in density.

In our day to day life in many cases the changes in pressure and temperature are sufficiently small that the changes in density are negligible. In that cases the flow are modeled as incompressible. Incompressibility is expressed by saying that the density ρ of a fluid parcel does not change as it moves in the flow field *i.e.* incompressible means that their volume do not change when the pressure changes or the density of every particle of a fluid remains constant following the motion. Although all known liquids are compressible but for the practical purposes they are regarded as incompressible fluids.

Mathematically, $\frac{D\rho}{Dt} = 0$

Viscous and Non-viscous fluid

An infinitesimal fluid element is acted upon by two of types of forces, namely, body forces and surface forces. The former is a type of force which is proportional to the mass or possibly the volume of the body on which it acts while the later is one which acts on a surface element and is proportional to the surface area.

Suppose that the fluid element be enclosed by the surface S. Let P be an arbitrary point of S and dS be the surface element around P. Then the surface force on dS is not in general in the direction of normal at P to dS. Hence the force may be resolved into components, one normal and other tangential to the area dS. The normal force per unit area is said to be the normal stress or pressure while the tangential force per unit area is said to be the shearing stress. A fluid is said to be viscous when normal as well as shearing stresses exist. On the other hand, a fluid is said to be inviscous when it does not exert any shearing stress, whether at rest or in motion. Due to shearing stress a viscous fluid produces resistance to the body moving through it as well as between the particles of the fluid itself. For example, water and air are treated inviscous fluids whereas syrup and heavy oil are treated as viscous fluid.

2.4 Free Convection

In the studies related to heat transfer, considerable effort has been directed towards the convective mode, in which the relative motion of the fluid provides an additional mechanism for the transfer of the energy and material, the latter being a more important consideration in case where mass transfer, due to a concentration difference occurs. Convection is inevitable coupled with the conductive mechanism, although the fluid motion modifies the transport process, the eventual transfer of energy from one fluid to another in its neighborhood is though condition. Also at the surface, the process is predominantly that of condition because the relative fluid motion is brought to zero at the surface. A study of the convective heat transfer therefore involves the mechanism of conduction and sometimes those of radioactive process as well, coupled with those of fluid flow. This makes the study of this mode of heat or mass transfer very complex, although its importance in technology and in nature can hardly be eager. In the general case of thermal boundary layers, the velocity field and the temperature field manually interact, which means that the temperature distribution depends on the velocity distribution. Conversely, the velocity distribution depends on the temperature distribution. In the special case when buoyancy forces may be disregarded, and when properties of the fluid may assumed to be independent of temperature, mutual interaction cases, and the velocity field no longer depends on the temperature field, although the converse dependence of the temperature field on the velocity field will still persists. This happens to large velocities (large Reynolds Number) and small temperature differences, such flows being termed forced. The process of heat transfer in such flow is described as forced convection. Flows in which buoyancy are dominant are called natural, the respective heat transfer being known as natural convection. This case occurs at every small velocities of motion in presence of large temperature difference.

2.5 MHD and Heat Transfer

With the advent of hypersonic flight, the field of MHD, as define above, which has attracted the interest of aero dynamics and associated largely with liquid metal pumping. It is possible to alter the flow and the heat transfer around high velocity vehicles provided that the air is sufficiently ionized. Furthermore, the invention of high temperature facilities such as the shock tube plasma jet has provided laboratory source of following ionized gas, which provided an insentive for the study of plasma accelerators and generators. As a result of this, many of the classical problems of fluid mechanics have been reinvestigated. Some of these analyses awake out of the natural tendency of science to search a new subject. In this case it was the academic problem of solving the equations of fluid mechanics with a new body force and another source of dissipation in the energy equation. Some time there were no practical applications for these results. As for example, natural convection MHD flows have been of interest to the engineering community only since the investigations, directly applicable to the problems in geophysics and astrophysics. But it was in the field of aerodynamic heating that the largest interest was awaked.

2.6 Heat and Mass transfer

Combined heat and mass transfer problems are of importance in many processes and have therefore received a considerable amount of attention. In many mass transfer processes, heat transfer considerations arise owing to chemical reaction and are often due to the natural of the process. In process such as drying, evaporation at the surface water, energy transfer in a wet cooling tower and the flow in a desert cooler, the interest lies in the determination of the total energy transfer, although in process such as drying, the interest lies mainly in the overall mass transfer for moisture removal. Nature convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, in many mixtures in the absence of an externally induced flow and many chemical processing systems. In many processes such as the curing of the plastic, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp insulated cables etc. the combined buoyancy mechanisms arise and the total energy and the material transfer resulting from the combined mechanisms has to be determined.

2.7 Porous Medium

A porous medium (or a porous material) is a material containing pores (voids). The skeletal portion of the material is often called the "matrix" or "frame". The pores are typically filled with a fluid (liquid or gas). The skeletal material is usually a solid but structures like foams are often also usefully analyzed using concept of porous media.

A porous medium is most often characterized by its porosity. Other properties of the medium (*e.g.*, permeability, strength, electrical) can sometimes be derived from the respective properties of its constituents (solid matrix and fluid) and the media porosity and pores structure, but such a derivation is usually complex. Even the concept of porosity is only straight forward for a poroelastic medium.

Often both the solid matrix and the pore network (also known as the pore space) are continuous, so as to form two interpenetrating continua such as in a sponge. However, there is also a concept of closed porosity and effective porosity, *i.e.*, the pore space accessible to flow.

Many natural substances such as rocks and soil (*e.g.*, aquifers, petroleum reservoirs), zealots, biological tissues (*e.g.* bones, wood, cork), and manmade materials such as cements and ceramics can be considered as porous media. Many of their important properties can only be rationalized by considering them to be porous media.

The concept of porous media is used in many areas of applied science and engineering: filtration, mechanics (acoustics, geomechanics, soil mechanics, rock mechanics), engineering (petroleum engineering, bio-remediation, construction engineering), geosciences (hydrogeology, petroleumgeology, geophysics), biology and biophysics, material science, etc. Fluid flow through porous media is a subject of most common interest and has emerged a separate field of study. The study of more general behavior of porous media involving deformation of the solid frame is called poromechanics.

2.8 Viscosity

Viscosity is one kind of property of a real fluid which generates shear stress between two fluid elements. An infinitesimal fluid element is acted upon by two types of forces, namely body forces and surface forces.

Let P be an arbitrary point of S and *ds* be the surface element around P. Then the surface force on *ds* is in general not in the direction of normal at P to *ds*. Hence the force may be resolved into two components, one normal and other tangential to the area *ds*. The normal force per unit area is said to be the normal stress or pressure while the tangential force per unit area is said to be the shearing stress.

We know that the flow of water and oil is much easier than syrup and heavy oil. This illustrates the existence of a property in the fluid, which controls its rate of flow. This property of fluids is said to be viscosity or internal friction.

A fluid is said to be viscous when normal stress as well as shearing stress exist, otherwise it is called inviscid fluid.

2.9 Newton's Law of Viscosity

A relationship between the shear stress and the velocity field was first stated by Newton. He concluded that the internal friction between two adjacent fluid particles should be independent of the normal pressure between them but proportional to the difference in their velocities.

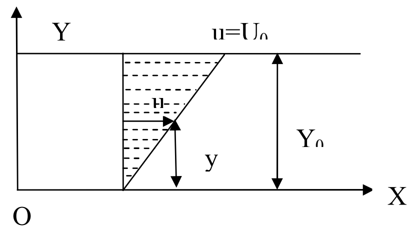

Figure 2.9.1 Physical configuration of viscosity

Consider a layer between two parallel plates lying at a distance y_0 from each other. Let the lower plate be fixed, while the upper is moving with a velocity u_0 uniformly and parallel to the lower one. A resistance F is experienced, and to a first approximation is given by the formula

$$F = \mu A_0 \left(\frac{U_0}{Y_0} \right) \tag{2.9.1}$$

where, A_0=Area of upper plate.

μ =Constant of proportionality which is called the coefficient of viscosity or the coefficient of dynamic viscosity.

The real fluids have no velocity at the wall for this reason they cannot slip at the boundary wall. This is known as no slip condition.

Since there is no slip on the wall for real fluids, the velocity u of a layer at a distance y from the lower plate is given by

$$u = U_0\left(\frac{y}{Y_0}\right) \text{ so that } \frac{U_0}{Y_0} = \frac{u}{y} \qquad (2.9.2)$$

So from equations (2.9.1) and (2.9.2) we get,

$$F = \mu A_0\left(\frac{u}{y}\right)$$

$$\Rightarrow \frac{F}{A_0} = \mu\left(\frac{u}{y}\right)$$

$$\Rightarrow T = \mu\left(\frac{u}{y}\right) \qquad (2.9.3)$$

where, $T = F/A_0$ is the friction or tangential force per unit area or the shear stress.

u/y=velocity gradient.

du/dy = differential form of velocity gradient.

Then equation (2.9.3) becomes

$$T = \mu\left(\frac{du}{dy}\right) \qquad (2.9.4)$$

This is known as the Newton's law of viscosity. Equation (2.9.4) can be regarded as the definition of viscosity.

Fluids which obey Newton's law of viscosity are known as Newtonian fluids. For example water, air and mercury are all Newtonian fluids.

Fluids which do not obey Newton's law of viscosity are known as non-Newtonian fluids. The fluids in which the shear stress is not proportional to the velocity gradient are called non-Newtonian fluids. For example paints, coal tar and polymer solutions are all non-Newtonian fluids.

2.10 Body and Surface Forces

There are two types of forces acting on a fluid element. They are body forces and surface forces. The body forces are distributed throughout the volume of the body and these are usually expressed as 'force per unit mass of the element'. For examples gravity and inertia forces. Moreover such forces may arise from other physical reason also such as electric and magnetic.

Surface forces are due to the action of surrounding fluid on the element under consideration through direct contact. Thus it is a boundary or surface action. These forces are expressed as ‘force per unit surface area of the element’.

2.11 Strain

Strain may be defined as a non-dimensional deformation which measures the change of relative positions of the parts of a body under any cause. Strain can be divided into the two steps. They are normal strain and shearing strain.

The ratio of the change in length to the original length of a linear element is known as the normal strain.

It is measured in terms of the change in the angle between two linear elements from the unstrained state to the strained state.

2.12 Hall Current

In the case of an electrically conducting rotating gas at low pressure, there has an interaction of the magnetic field with the electric field of both the electrons and the ionized atoms of the gas. If the magnetic field is perpendicular to the electric field, a current is induced in the conductive rotating gas whose direction is perpendicular to the both the electric field and the magnetic field. This current is called Hall current and is induced by a phenomenon known as Hall Effect.

The Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. The Hall effect was discovered in 1879 by **Edwin Herbert Hall** while he was working on his doctoral degree Johns Hopkins University in Baltimore, Maryland. His measurements of the tiny effect produced in the apparatus he was an experimental tour de force, accomplished 18 years before the electrons was discovered.

The Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and the applied magnetic field. It is a characteristic of the material from which the conductor is made, since its value depends on the type, number, and properties of the charge carries that constitute the current. The Hall effect in an ionized gas (plasma) is significantly different from the Hall effect in solids (where the Hall parameter is always very inferior to unity).

Mathematically, Hall current $m = w_e t_e, \quad w_e = \frac{eB_0}{m_e}$

where w_e is electronic frequency, e is charge of electron, B_0 is magnetic induction, m_e is mass of electron and t_e is electronic collision time

2.13 Some Definitions

Schmidt Number (*Sc*)

This the ratio of the viscous diffusivity to the chemical molecular diffusivity and is defined as

$$Sc = \frac{\text{Viscous diffusivity}}{\text{Chemical molecular diffusivity}} = \frac{\upsilon}{D}$$

Grashof Number (*Gr*)

This is denoted by Gr and defined as $Gr = \frac{g\beta\upsilon\left(\bar{T}_w - \bar{T}_\infty\right)}{v_0^3}$ and is a measure of the relative importance of the buoyancy forces and viscous forces.

Modified Grashof Number (*Gm*)

This is denoted by Gm and defined as $Gm = \frac{g\bar{\beta}\upsilon\left(\bar{C}_W - \bar{C}_\infty\right)}{v_0^3}$

Magnetic Parameter (*M*)

This is obtained from the ratio of the magnetic force to the inertia force and is defined as $M = \frac{\text{Magnetic force}}{\text{Inertia force}} = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2}$

Prandtl Number (*Pr*)

The Prandrtl number is the ratio of kinematic viscosity to thermal diffusivity and may be written as follows

$$Pr = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\upsilon}{\frac{\kappa}{\rho C_p}} = \frac{\mu C_p}{\kappa}$$

where C_p is the specific heat at constant pressure and   is the thermal conductivity.

The value of $\upsilon = \frac{\mu}{\rho}$ shows the effect of viscosity of the fluid. The smaller the value of υ is, the narrower is the region which is affected by viscosity and which is known as the boundary layer region when   is very small. The value of $\frac{\kappa}{\rho C_p}$ shows the thermal diffusivity due to heat conduction. The smaller the value of $\frac{\kappa}{\rho C_p}$ is, the narrower is the region which is affected by the heat conduction and which is known as thermal boundary layer when $\frac{\kappa}{\rho C_p}$ is small. Thus the Prandtl number shows the relative importance of heat conduction and viscosity of a fluid. Evidently, the value of *Pr* varies from fluid to fluid. At 20^0C the value of *Pr* for air is 0.71 (approx), for water it is 7.0(approx), similarly for mercury it is 0.044 (approx) but for high viscous fluid it may be very large for example at 20^0C the value of *Pr* is 7250(approx).

Eckert Number (*Ec*)

The Eckert number *Ec* is defined by $Ec = \frac{v_0^{\;2}}{C_P\left(\overline{T}_w - \overline{T}_\infty\right)}$

where $\left(\overline{T}_w - \overline{T}_\infty\right)$ is the temperature difference between the wall and the fluid at a large distance from the body.

Shear Stress ( )

The dimensionless skin-friction coefficient is generally known as the shear stress at the plate and is defined as follows:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Nusselt Number (*Nu*)

The dimensionless rate of heat transfer is known as the Nusselt number and is defined as follows:

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

Sherwood Number (*Sh*)

The dimensionless coefficient of mass transfer is known as the Sherwood number and is defined as follows:

$$S_h = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

Chapter 3

Governing Equations and Solution Procedure

3.1 Governing Equations

Consider the two-dimensional flow of an electrically conducting, viscous, incompressible, radiating, fluid of density   through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface. Figure 3.1.1 shows the physical model where the *x*-axis is taken along the vertical and *y* axis is horizontal perpendicular to the plate. A uniform magnetic field B_0 is applied normally to the flow region.

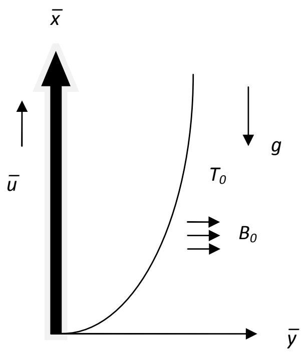

Figure 3.1.1 The physical co-ordinate system

Therefore the governing equations describing the model proposed in the study are

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{3.1.1}$$

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta\left(\bar{T} - \bar{T}_\infty\right) + g\bar{\beta}\left(\bar{C} - \bar{C}_\infty\right) - \frac{\sigma B_0^{\,2}}{\rho\left(1+m^2\right)}\bar{u} - \frac{\upsilon}{\bar{k}}\bar{u} \tag{3.1.2}$$

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\upsilon}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} \tag{3.1.3}$$

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - Kc\left(\bar{C} - \bar{C}_\infty\right) \tag{3.1.4}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{lll} \bar{y} = 0 & : \ \bar{u} = 0, & \bar{T} = \bar{T}_w, \quad \bar{C} = \bar{C}_w \\ \bar{y} \to \infty & : \ \bar{u} \to 0, & \bar{T} \to \bar{T}_\infty, \quad \bar{C} \to \bar{C}_\infty \end{array}\right\} \tag{3.1.5}$$

Here u and v are the velocity components along the x and y- directions respectively, T is the fluid temperature; T_w is the temperature of the fluid at the plate, T is the fluid temperature far away from the plate, g is the acceleration due to gravity, is the thermal conductivity, ρ is the density of the fluid, C_p is the specific heat at constant pressure, is the electrical conductivity, D is the chemical molecular diffusivity, v_0 is the uniform velocity, C is the concentration of species, C_w is the mean concentration, C is the concentration of species for uniform flow, B_0 is the uniform applied magnetic field, is the density, υ is the kinematic viscosity, is the coefficient of thermal expansion and $\bar{\beta}$ is the coefficient of thermal expansion with concentration and the other symbols have their usual meaning.

3.2 Calculation Technique

Many physical phenomena in applied science and engineering when formulated into mathematical models fall into a category of systems known as non-linear coupled partial differential equations. Most of these problems can be formulated as second order partial differential equations. A system of non-linear coupled partial differential equations with the boundary conditions is very difficult to solve analytically. The governing equations of our problem contain a system of partial differential equations which are transformed by usual transformations into a non-dimensional system of non-linear coupled partial differential equations with boundary conditions. However, a great deal of insight as to the flow behavior can be obtained if we adopt a perturbation method. Hence the solution of our problem would be based on perturbation methods.

According to perturbation techniques, the solution is represented by the first few terms of an asymptotic expansion, usually not more than three terms. The expansion may be carried out in terms of a parameter (small or large) which appears naturally in the equations, or which may be artificially introduced for convenience. Such expansions are called parameter perturbations. The difficulty other than complexity is that perturbation methods are ultimately series solutions in a small parameter.

3.3 Formulation of the Problem

To make dimensionless the governing equations from (3.1.1) to (3.1.4) under the boundary conditions (3.1.5) we now introduce the following dimensionless quantities.

$$\left.\begin{array}{l} u=\dfrac{1}{v_0}\bar{u},\ y=\dfrac{v_0}{\upsilon}\bar{y},\ \theta=\dfrac{\bar{T}-\bar{T}_\infty}{\bar{T}_W-\bar{T}_\infty},\ \phi=\dfrac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},\ F=\dfrac{4I_1\upsilon^2}{\kappa v_0^{\,2}} \\ Gr=\dfrac{g\beta\upsilon\left(\bar{T}_w-\bar{T}_\infty\right)}{v_0^{\,3}},\ Gm=\dfrac{g\bar{\beta}\upsilon\left(\bar{C}_W-\bar{C}_\infty\right)}{v_0^{\,3}},\ \Pr=\dfrac{\mu C_p}{\kappa},\ Sc=\dfrac{\upsilon}{D} \\ Kc=\dfrac{\upsilon}{v_0^{\,2}}\bar{K}c,\ Ec=\dfrac{v_0^{\,2}}{C_P\left(\bar{T}_w-\bar{T}_\infty\right)},\ M=\dfrac{\sigma B_0^{\,2}\upsilon}{\rho v_0^{\,2}},\ k=\dfrac{v_0^{\,2}}{\upsilon^2}\bar{k} \end{array}\right\} \tag{3.3.1}$$

Equation (3.1.1) implies,

$$\bar{v}=-v_0\,(say) \tag{3.3.2}$$

In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self absorption, but it does absorb radiation emitted by the boundaries. Mahaptra *et al.* [25] showed that in the optically thick limit for a non gray near equilibrium as

$$\frac{\partial q_r}{\partial \bar{y}}=4I_1\left(\bar{T}-\bar{T}_\infty\right) \tag{3.3.3}$$

Now, $\dfrac{\partial \bar{u}}{\partial \bar{y}}=\dfrac{\partial \bar{u}}{\partial u}\dfrac{\partial u}{\partial \bar{y}}=\dfrac{\partial \bar{u}}{\partial u}\dfrac{\partial y}{\partial \bar{y}}\dfrac{\partial u}{\partial y}=v_0\dfrac{v_0}{\upsilon}\dfrac{\partial u}{\partial y}=\dfrac{v_0^{\,2}}{\upsilon}\dfrac{\partial u}{\partial y}$

$$\therefore \frac{\partial^2\bar{u}}{\partial \bar{y}^2}=\frac{\partial}{\partial \bar{y}}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)=\frac{\partial}{\partial y}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)\frac{\partial y}{\partial \bar{y}}=\frac{\partial}{\partial y}\left(\frac{v_0^{\,2}}{\upsilon}\frac{\partial u}{\partial y}\right)\frac{\partial y}{\partial \bar{y}}=\frac{v_0^{\,2}}{\upsilon}\frac{\partial^2 u}{\partial y^2}\frac{v_0}{\upsilon}=\frac{v_0^{\,3}}{\upsilon^2}\frac{\partial^2 u}{\partial y^2}$$

So we get from (3.1.2)

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta\left(\bar{T}-\bar{T}_\infty\right) + g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right) - \frac{\sigma B_0^{\ 2}}{\rho\left(1+m^2\right)}\bar{u} - \frac{\upsilon}{\bar{k}}\bar{u}$$

$$\Rightarrow -v_0\frac{v_0^{\ 2}}{\upsilon}\frac{\partial u}{\partial y} = \upsilon\frac{v_0^{\ 3}}{\upsilon^2}\frac{\partial^2 u}{\partial y^2} + g\beta\left(\bar{T}-\bar{T}_\infty\right) + g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right) - \frac{\sigma B_0^{\ 2}}{\rho\left(1+m^2\right)}v_0 u - \frac{v_0^{\ 3}}{\upsilon k}u$$

$$\Rightarrow -\frac{v_0^{\ 3}}{\upsilon}\frac{\partial u}{\partial y} = \frac{v_0^{\ 3}}{\upsilon}\frac{\partial^2 u}{\partial y^2} + g\beta\left(\bar{T}-\bar{T}_\infty\right) + g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right) - \frac{\sigma B_0^{\ 2}}{\rho\left(1+m^2\right)}v_0 u - \frac{v_0^{\ 3}}{\upsilon k}u$$

$$\Rightarrow -\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{g\beta\upsilon\left(\bar{T}-\bar{T}_\infty\right)}{v_0^{\ 3}} + \frac{g\bar{\beta}\upsilon\left(\bar{C}-\bar{C}_\infty\right)}{v_0^{\ 3}} - \frac{\sigma B_0^{\ 2}\upsilon}{\rho v_0^{\ 2}\left(1+m^2\right)}u - \frac{u}{k}$$

$$\Rightarrow -\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{g\beta\upsilon\left(\bar{T}_w-\bar{T}_\infty\right)}{v_0^{\ 3}}\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty} + \frac{g\bar{\beta}\upsilon\left(\bar{C}_w-\bar{C}_\infty\right)}{v_0^{\ 3}}\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty} - \frac{M}{1+m^2}u - \frac{u}{k}$$

$$\Rightarrow -\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi - \left(\frac{M}{1+m^2}+\frac{1}{k}\right)u$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - Nu = -Gm\phi - Gr\theta\,,\ N = \frac{M}{1+m^2}+\frac{1}{k}$$

Again we have

$$\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\partial \bar{T}}{\partial \theta}\frac{\partial \theta}{\partial \bar{y}} = \frac{\partial \bar{T}}{\partial \theta}\frac{\partial y}{\partial \bar{y}}\frac{\partial \theta}{\partial y} = \frac{v_0}{\upsilon}\left(\bar{T}_w-\bar{T}_\infty\right)\frac{\partial \theta}{\partial y}$$

$$\therefore \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} = \frac{\partial}{\partial \bar{y}}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{\partial}{\partial y}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)\frac{\partial y}{\partial \bar{y}} = \frac{\partial}{\partial y}\left\{\frac{v_0}{\upsilon}\left(\bar{T}_w-\bar{T}_\infty\right)\frac{\partial \theta}{\partial y}\right\}\frac{\partial y}{\partial \bar{y}} = \frac{v_0^{\ 2}}{\upsilon^2}\left(\bar{T}_w-\bar{T}_\infty\right)\frac{\partial^2 \theta}{\partial y^2}$$

Also $\dfrac{\kappa}{\rho C_p} = \dfrac{\mu C_p}{\Pr}\dfrac{1}{\frac{\mu}{\upsilon}C_p} = \dfrac{\upsilon}{\Pr}$

and $\dfrac{1}{\rho C_p}\dfrac{\partial q_r}{\partial \bar{y}} = \dfrac{\upsilon}{\kappa\Pr}\dfrac{F\kappa v_0^{\ 2}}{\upsilon^2}\left(\bar{T}-\bar{T}_\infty\right) = \dfrac{v_0^{\ 2}}{\upsilon}\dfrac{F}{\Pr}\left(\bar{T}-\bar{T}_\infty\right)$

Thus from (3.1.3) we get

$$-v_0\frac{v_0}{\upsilon}\left(\bar{T}_w-\bar{T}_\infty\right)\frac{\partial \theta}{\partial y} = \frac{\upsilon}{\Pr}\frac{v_0^{\ 2}}{\upsilon^2}\left(\bar{T}_w-\bar{T}_\infty\right)\frac{\partial^2 \theta}{\partial y^2} + \frac{\upsilon}{C_p}\frac{v_0^{\ 4}}{\upsilon^2}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{v_0^{\ 2}}{\upsilon}\frac{F}{\Pr}\left(\bar{T}-\bar{T}_\infty\right)$$

$$\Rightarrow \frac{-v_0^{\,2}}{\upsilon}\left(\overline{T}_w-\overline{T}_\infty\right)\frac{\partial\theta}{\partial y}=\frac{v_0^{\,2}}{\upsilon}\frac{1}{\Pr}\left(\overline{T}_w-\overline{T}_\infty\right)\frac{\partial^2\theta}{\partial y^2}+\frac{v_0^{\,4}}{\upsilon}\frac{1}{C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{v_0^{\,2}}{\upsilon}\frac{F}{\Pr}\left(\overline{T}-\overline{T}_\infty\right)$$

$$\Rightarrow -\Pr\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2}+\frac{v_0^{\,2}}{C_p\left(\overline{T}_w-\overline{T}_\infty\right)}\Pr\left(\frac{\partial u}{\partial y}\right)^2-F\frac{\overline{T}-\overline{T}_\infty}{\overline{T}_w-\overline{T}_\infty}$$

$$\Rightarrow \frac{\partial^2\theta}{\partial y^2}+\Pr\frac{\partial\theta}{\partial y}-F\theta=-Ec\Pr\left(\frac{\partial u}{\partial y}\right)^2$$

And finally,

$$\frac{\partial\overline{C}}{\partial\overline{y}}=\frac{\partial\overline{C}}{\partial\phi}\frac{\partial\phi}{\partial\overline{y}}=\frac{\partial\overline{C}}{\partial\phi}\frac{\partial y}{\partial\overline{y}}\frac{\partial\phi}{\partial y}=\left(\overline{C}_w-\overline{C}_\infty\right)\frac{v_0}{\upsilon}\frac{\partial\phi}{\partial y}=\frac{v_0}{\upsilon}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial\phi}{\partial y}$$

$$\Rightarrow \frac{\partial^2\overline{C}}{\partial\overline{y}^2}=\frac{\partial}{\partial\overline{y}}\left(\frac{\partial\overline{C}}{\partial\overline{y}}\right)=\frac{\partial}{\partial y}\left(\frac{\partial\overline{C}}{\partial\overline{y}}\right)\frac{\partial y}{\partial\overline{y}}=\frac{\partial}{\partial y}\left\{\frac{v_0}{\upsilon}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial\phi}{\partial y}\right\}\frac{\partial y}{\partial\overline{y}}=\frac{v_0^{\,2}}{\upsilon^2}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial^2\phi}{\partial y^2}$$

Therefore, we have from (3.1.4)

$$-v_0\frac{v_0}{\upsilon}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial\phi}{\partial y}=D\frac{v_0^{\,2}}{\upsilon^2}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial^2\phi}{\partial y^2}-\overline{K}c\left(\overline{C}-\overline{C}_\infty\right)$$

$$\Rightarrow -\frac{v_0^{\,2}}{\upsilon}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial\phi}{\partial y}=D\frac{v_0^{\,2}}{\upsilon^2}\left(\overline{C}_w-\overline{C}_\infty\right)\frac{\partial^2\phi}{\partial y^2}-\overline{K}c\left(\overline{C}-\overline{C}_\infty\right)$$

$$\Rightarrow -\frac{\upsilon}{D}\frac{\partial\phi}{\partial y}=\frac{\partial^2\phi}{\partial y^2}-\frac{\overline{K}c}{D}\frac{\upsilon^2}{v_0^{\,2}}\frac{\overline{C}-\overline{C}_\infty}{\overline{C}_w-\overline{C}_\infty}$$

$$\Rightarrow -Sc\frac{\partial\phi}{\partial y}=\frac{\partial^2\phi}{\partial y^2}-\frac{\upsilon}{D}\frac{\overline{K}c\upsilon}{v_0^{\,2}}\phi$$

$$\Rightarrow \frac{\partial^2\phi}{\partial y^2}+Sc\frac{\partial\phi}{\partial y}-KcSc\phi=0$$

Thus the non-dimensional form of the governing equations (3.1.2), (3.1.3) and (3.1.4) are respectively as follows:

$$u''+u'-Nu=-Gm\phi-Gr\theta \qquad (3.3.4)$$

where $N=\dfrac{M}{1+m^2}+\dfrac{1}{k}$

$$\theta''+\Pr\theta'-F\theta=-\Pr Ecu'^2 \qquad (3.3.5)$$

$$\phi'' + Sc\phi' - KcSc\phi = 0 \tag{3.3.6}$$

where dashes denote differentiation with respect to y.

The corresponding boundary conditions (3.1.5) in non-dimensional forms are:

$$\left.\begin{array}{l} y = 0 \quad : u = 0, \quad \theta = 1, \quad \phi = 1 \\ y \to \infty \ : u \to 0, \ \theta \to 0, \ \phi \to 0 \end{array}\right\} \tag{3.3.7}$$

3.4 Method of Solution

To solve the equations (3.3.4), (3.3.5) and (3.3.6) with boundary conditions (3.3.7), we use the following simple perturbation. The governing equations (3.3.4), (3.3.5) and (3.3.6) are expanded in power of Eckert number Ec(<<1) so we consider Ec as a perturbation quantity. Also we consider a second order correction of it.

$$\left.\begin{array}{l} u(y) = u_0(y) + Ecu_1(y) + O\left(Ec^2\right) \\ \theta(y) = \theta_0(y) + Ec\theta_1(y) + O\left(Ec^2\right) \\ \phi(y) = \phi_0(y) + Ec\phi_1(y) + O\left(Ec^2\right) \end{array}\right\} \tag{3.4.1}$$

Substituting (3.4.1) in equations (3.3.4), (3.3.5) and (3.3.6) we get:

$$(3.3.4) \Rightarrow u_0'' + Ecu_1'' + O\left(Ec^2\right) + u_0' + Ecu_1' + O\left(Ec^2\right) - N\left(u_0 + Ecu_1 + O\left(Ec^2\right)\right)$$
$$= -Gm\left(\phi_0 + Ec\phi_1 + O\left(Ec^2\right)\right) - Gr\left(\theta_0 + Ec\theta_1 + O\left(Ec^2\right)\right)$$
$$\Rightarrow \left(u_0'' + u_0' - Nu_0\right) + Ec\left(u_1'' + u_1' - Nu_1\right) + O\left(Ec^2\right)$$
$$= -Gm\phi_0 - Gr\theta_0 - Ec\left(Gm\phi_1 + Gr\theta_1\right) + O\left(Ec^2\right) \tag{3.4.2}$$

Again,

$$(3.3.5) \Rightarrow \theta_0'' + Ec\theta_1'' + O\left(Ec^2\right) + \Pr\left(\theta_0' + Ec\theta_1' + O\left(Ec^2\right)\right) - F\left(\theta_0 + Ec\theta_1 + O\left(Ec^2\right)\right)$$
$$= -\Pr Ec\left(u_0' + Ecu_1' + O\left(Ec^2\right)\right)^2$$
$$\Rightarrow \left(\theta_0'' + \Pr\theta_0' - F\theta_0\right) + Ec\left(\theta_1'' + \Pr\theta_1' - F\theta_1\right) + O\left(Ec^2\right) = -Ec\Pr u_0'^2 + O\left(Ec^2\right) \tag{3.4.3}$$

And,

$$(3.3.6) \Rightarrow \phi_0'' + Ec\phi_1'' + O\left(Ec^2\right) + Sc\left(\phi_0' + Ec\phi_1' + O\left(Ec^2\right)\right) - KcSc\left(\phi_0 + Ec\phi_1 + O\left(Ec^2\right)\right) = 0$$

$$\Rightarrow \phi_0'' + Sc\phi_0' - KcSc\phi_0 + Ec\left(\phi_1'' + Sc\phi_1' - KcSc\phi_1\right) + O\left(Ec^2\right) = 0 \tag{3.4.4}$$

Now equating the coefficients of Ec^0 and Ec^1 and neglecting those of Ec^2 and higher powers, form the equations (3.4.2), (3.4.3) and (3.4.4) we have

$$u_0'' + u_0' - Nu_0 = -Gm\phi_0 - Gr\theta_0 \tag{3.4.5}$$

$$u_1'' + u_1' - Nu_1 = -Gm\phi_1 - Gr\theta_1 \tag{3.4.6}$$

$$\theta_0'' + \Pr\theta_0' - F\theta_0 = 0 \tag{3.4.7}$$

$$\theta_1'' + \Pr\theta_1' - F\theta_1 = -\Pr u_0'^2 \tag{3.4.8}$$

$$\phi_0'' + Sc\phi_0' - ScKc\phi_0 = 0 \tag{3.4.9}$$

$$\phi_1'' + Sc\phi_1' - ScKc\phi_1 = 0 \tag{3.4.10}$$

subject to the boundary conditions:

$$\left.\begin{array}{ll} y = 0 & : u_0 = 0,\ \ u_1 = 0,\ \ \theta_0 = 1,\ \ \theta_1 = 0,\ \ \phi_0 = 1,\ \ \phi_1 = 0 \\ y \to \infty & : u_0 \to 0,\ u_1 \to 0,\ \theta_0 \to 0,\ \theta_1 \to 0,\ \phi_0 \to 0,\ \phi_1 \to 0 \end{array}\right\} \tag{3.4.11}$$

Now we have to solve the above equations (3.4.5) to (3.4.10) using boundary conditions (3.4.11).

To find the solution of the above equations let us consider, $\phi_0(y) = e^{my}$ (where $e^{my} \neq 0$) as a trial solution of equation (3.4.9)

$$\therefore \phi_0'(y) = me^{my}$$

and $\phi_0''(y) = m^2 e^{my}$

Using above we have from equation (3.4.9)

$$m^2 e^{my} + Scme^{my} - ScKce^{my} = 0$$

$$\Rightarrow \left(m^2 + Scm - ScKc\right)e^{my} = 0$$

$\therefore m^2 + Scm - ScKc = 0$ since $e^{my} \neq 0$

which is called the auxiliary equation.

Now values of m are $m = \frac{1}{2}\left\{-Sc \pm \sqrt{Sc^2 + 4ScKc}\right\}$

Therefore the general solution will be

$$\phi_0 = C_1 e^{\frac{1}{2}\left\{-Sc+\sqrt{Sc^2+4ScKc}\right\}y} + C_2 e^{-\frac{1}{2}\left\{Sc+\sqrt{Sc^2+4ScKc}\right\}y} \tag{3.4.12}$$

To get the values of C_1 and C_2, we use the boundary conditions

$\phi_0 = 1$ on $y = 0$
$\phi_0 \to 0$ as $y \to \infty$

then we obtain C_1=0 and C_2=1

$$\therefore (3.4.12) \Rightarrow \phi_0 = e^{-k_1 y} \tag{3.4.13}$$

where $k_1 = \frac{1}{2}\left\{Sc + \sqrt{Sc^2 + 4ScKc}\right\}$

Again, let, a solution of equation (3.4.10) is $\phi_1(y) = e^{my}$ (where $e^{my} \neq 0$)

$\therefore \phi_1'(y) = me^{my}$

and $\phi_1''(y) = m^2 e^{my}$

so from (3.4.10) we get

$m^2 e^{my} + Scme^{my} - ScKce^{my} = 0$

$\Rightarrow \left(m^2 + Scm - ScKc\right)e^{my} = 0$

$\therefore m^2 + Scm - ScKc = 0$ since $e^{my} \neq 0$

which is called the auxiliary equation.

The values of m are, $m = \frac{1}{2}\left\{-Sc \pm \sqrt{Sc^2 + 4ScKc}\right\}$

Thus the general solution of (3.4.10) is

$$\phi_1 = C_1 e^{\frac{1}{2}\left\{-Sc+\sqrt{Sc^2+4ScKc}\right\}y} + C_2 e^{-\frac{1}{2}\left\{Sc+\sqrt{Sc^2+4ScKc}\right\}y} \tag{3.4.14}$$

Using the boundary conditions

$\phi_1 = 0 \quad \text{on} \quad y = 0$
$\phi_1 \to 0 \quad \text{as} \quad y \to \infty$

we get C_1=0 and C_2=0

$\therefore (3.4.14) \Rightarrow \phi_1 = 0$ (3.4.15)

To solve the equation (3.4.7), let, $\theta_0(y) = e^{my}$ (where $e^{my} \neq 0$) is a probable solution of it.

$\therefore \theta_0'(y) = me^{my}$

and $\theta_0''(y) = m^2 e^{my}$

So we have from (3.4.7) the auxiliary equation (A.E.) :

$m^2 e^{my} + \Pr m e^{my} - F e^{my} = 0$

$\Rightarrow \left(m^2 + \Pr m - F\right) e^{my} = 0$

$\therefore m^2 + \Pr m - F = 0$ since $e^{my} \neq 0$

$\Rightarrow m = \frac{1}{2}\left\{-\Pr \pm \sqrt{\Pr^2 + 4F}\right\}$

So the general solution of (3.4.7) is

$$\theta_0 = C_1 e^{\frac{1}{2}\left\{-\Pr + \sqrt{\Pr^2 + 4F}\right\}y} + C_2 e^{-\frac{1}{2}\left\{\Pr + \sqrt{\Pr^2 + 4F}\right\}y} \quad (3.4.16)$$

Using boundary conditions

$\theta_0 = 1 \quad \text{on} \quad y = 0$
$\theta_0 \to 0 \quad \text{as} \quad y \to \infty$

we get C_1=0 and C_2=1

$\therefore (3.4.16) \Rightarrow \theta_0 = e^{-k_2 y}$ (3.4.17)

where $k_2 = \frac{1}{2}\left\{\Pr + \sqrt{\Pr^2 + 4F}\right\}$

Again $(3.4.5) \Rightarrow u_0'' + u_0' - N u_0 = -Gm\phi_0 - Gr\theta_0$

$\Rightarrow u_0'' + u_0' - N u_0 = -Gm e^{-k_1 y} - Gr e^{-k_2 y}$ [by (3.4.13) and (3.4.13)] (3.4.18)

Similarly let $u_0(y) = e^{my}$ (where $e^{my} \neq 0$) is a solution of (3.4.18)

$\therefore u_0'(y) = me^{my}$

and $u_0''(y) = m^2 e^{my}$

So from (3.4.18), the auxiliary equation (A.E.):

$m^2 e^{my} + me^{my} - Ne^{my} = 0$

$\Rightarrow (m^2 + m - N)e^{my} = 0$

$\therefore m^2 + m - N = 0$ since $e^{my} \neq 0$

$\Rightarrow m = \frac{1}{2}\left\{-1 \pm \sqrt{1+4N}\right\}$

Then the complimentary function (C.F.) takes the form: $u_{o_c} = C_1 e^{\frac{1}{2}\left\{-1+\sqrt{1+4N}\right\}y} + C_2 e^{-\frac{1}{2}\left\{1+\sqrt{1+4N}\right\}y}$

Now particular integral (P.I.): $u_{0_p} = \frac{1}{D^2 + D - N}\left(-Gme^{-k_1 y} - Gre^{-k_2 y}\right)$

$$= -Gm\frac{1}{D^2 + D - N}e^{-k_1 y} - Gr\frac{1}{D^2 + D - N}e^{-k_2 y}$$

$$= -Gm\frac{1}{k_1^{\,2} - k_1 - N}e^{-k_1 y} - Gr\frac{1}{k_2^{\,2} - k_2 - N}e^{-k_2 y}$$

Thus the general solution is $u_o = u_{o_c} + u_{o_p}$

$$\Rightarrow u_o = C_1 e^{\frac{1}{2}\left\{-1+\sqrt{1+4N}\right\}y} + C_2 e^{-\frac{1}{2}\left\{1+\sqrt{1+4N}\right\}y} - Gm\frac{1}{k_1^{\,2} - k_1 - N}e^{-k_1 y} - Gr\frac{1}{k_2^{\,2} - k_2 - N}e^{-k_2 y} \qquad (3.4.19)$$

To get the values of the constants C_1 and C_2, we use the following boundary conditions

$u_0 = 0$ on $y = 0$
$u_0 \to 0$ as $y \to \infty$

Then we get C_1=0 and $C_2 = Gm\frac{1}{k_1^{\,2} - k_1 - N} + Gr\frac{1}{k_2^{\,2} - k_2 - N}$

Thus the general solution becomes

$$u_0 = k_3 e^{-k_1 y} + k_4 e^{-k_2 y} + k_5 e^{-k_6 y} \qquad (3.4.20)$$

where $k_3 = -\frac{Gm}{k_1^2 - k_1 - N}$, $k_4 = -\frac{Gr}{k_2^2 - k_2 - N}$, $k_5 = -k_3 - k_4$, $k_6 = \frac{1}{2}\left\{1+\sqrt{1+4N}\right\}$

Again for the solution of the equation (3.4.8) we have

$$\theta_1'' + \Pr\theta_1' - F\theta_1 = -\Pr u_0'^2$$

$$= -\Pr\left(-k_1k_3e^{-k_1y} - k_2k_4e^{-k_2y} - k_5k_6e^{-k_6y}\right)^2 \quad \text{[by (3.4.20)]}$$

$$= -\Pr\left[k_1^2k_3^2e^{-2k_1y} + k_2^2k_4^2e^{-2k_2y} + k_5^2k_6^2e^{-2k_6y} + 2k_1k_2k_3k_4e^{-(k_1+k_2)y}\right.$$

$$\left. + 2k_1k_3k_5k_6e^{-(k_1+k_6)y} + 2k_2k_4k_5k_6e^{-(k_2+k_6)y}\right]$$

$$\Rightarrow \theta_1'' + \Pr\theta_1' - F\theta_1 = -\Pr\left[k_7e^{-2k_1y} + k_8e^{-2k_2y} + k_9e^{-2k_6y} + k_{10}e^{-k_{11}y} + k_{12}e^{-k_{13}y} + k_{14}e^{-k_{15}y}\right] \qquad (3.4.21)$$

where, $k_7 = k_1^2k_3^2$, $k_8 = k_2^2k_4^2$, $k_9 = k_5^2k_6^2$, $k_{10} = 2k_1k_2k_3k_4$, $k_{11} = k_1 + k_2$,

$k_{12} = 2k_1k_3k_5k_6$, $k_{13} = k_1 + k_6$, $k_{14} = 2k_2k_4k_5k_6$, $k_{15} = k_2 + k_6$

Let $\theta_1(y) = e^{my}$ (where $e^{my} \neq 0$) is a probable solution of the equation (3.4.21)

$$\therefore \theta_1'(y) = me^{my}$$

and $\theta_1''(y) = m^2e^{my}$

So from the equation (3.4.21) the auxiliary equation (A.E.) is :

$$m^2e^{my} + \Pr me^{my} - Fe^{my} = 0$$

$$\Rightarrow \left(m^2 + \Pr m - F\right)e^{my} = 0$$

$$\Rightarrow m^2 + \Pr m - F = 0 \quad \text{since } e^{my} \neq 0$$

$$\therefore m = \frac{1}{2}\left\{-\Pr \pm \sqrt{\Pr^2 + 4F}\right\}$$

Complimentary function (C.F.) is: $\theta_{1_c} = C_1e^{\frac{1}{2}\left\{-\Pr+\sqrt{\Pr^2+4F}\right\}y} + C_2e^{-\frac{1}{2}\left\{\Pr+\sqrt{\Pr^2+4F}\right\}y}$

Now to get the solution of the non homogeneous part of (3.4.21) (P.I.):

$$\theta_{1_p} = \frac{1}{D^2 + \Pr D - F}\left\{-\Pr\left(k_7e^{-2k_1y} + k_8e^{-2k_2y} + k_9e^{-2k_6y} + k_{10}e^{-k_{11}y} + k_{12}e^{-k_{13}y} + k_{14}e^{-k_{15}y}\right)\right\}$$

$$= -\Pr\left[\frac{1}{D^2+\Pr D-F}k_7e^{-2k_1y}+\frac{1}{D^2+\Pr D-F}k_8e^{-2k_2y}+\frac{1}{D^2+\Pr D-F}k_9e^{-2k_6y}\right.$$

$$\left.+\frac{1}{D^2+\Pr D-F}k_{10}e^{-k_{11}y}+\frac{1}{D^2+\Pr D-F}k_{12}e^{-k_{13}y}+\frac{1}{D^2+\Pr D-F}k_{14}e^{-k_{15}y}\right]$$

$$= -\Pr\left[\frac{k_7}{4k_1^2-2\Pr k_1-F}e^{-2k_1y}+\frac{k_8}{4k_2^2-2\Pr k_2-F}e^{-2k_2y}+\frac{k_9}{4k_6^2-2\Pr k_6-F}e^{-2k_6y}\right.$$

$$\left.+\frac{k_{10}}{k_{11}^2-\Pr k_{11}-F}e^{-k_{11}y}+\frac{k_{12}}{k_{13}^2-\Pr k_{13}-F}e^{-k_{13}y}+\frac{k_{14}}{k_{15}^2-\Pr k_{15}-F}e^{-k_{15}y}\right]$$

Thus the general solution is $\theta_1=\theta_{1_c}+\theta_{1_p}$

$$\Rightarrow \theta_1 = C_1e^{\frac{1}{2}\left\{-\Pr+\sqrt{\Pr^2+4F}\right\}y}+C_2e^{-\frac{1}{2}\left\{\Pr+\sqrt{\Pr^2+4F}\right\}y}-\Pr\left[\frac{k_7}{4k_1^2-2\Pr k_1-F}e^{-2k_1y}\right.$$

$$+\frac{k_8}{4k_2^2-2\Pr k_2-F}e^{-2k_2y}+\frac{k_9}{4k_6^2-2\Pr k_6-F}e^{-2k_6y}+\frac{k_{10}}{k_{11}^2-\Pr k_{11}-F}e^{-k_{11}y}$$

$$\left.+\frac{k_{12}}{k_{13}^2-\Pr k_{13}-F}e^{-k_{13}y}+\frac{k_{14}}{k_{15}^2-\Pr k_{15}-F}e^{-k_{15}y}\right] \tag{3.4.22}$$

Using boundary conditions

$\theta_1=0$ on $y=0$

$\theta_1\to 0$ as $y\to\infty$

we have C_1=0

and $$C_2=\Pr\left[\frac{k_7}{4k_1^2-2\Pr k_1-F}+\frac{k_8}{4k_2^2-2\Pr k_2-F}+\frac{k_9}{4k_6^2-2\Pr k_6-F}+\frac{k_{10}}{k_{11}^2-\Pr k_{11}-F}\right.$$

$$\left.+\frac{k_{12}}{k_{13}^2-\Pr k_{13}-F}+\frac{k_{14}}{k_{15}^2-\Pr k_{15}-F}\right]$$

Therefore the required solution of the equation takes the following form (3.4.22)

$$\Rightarrow \theta_1 = k_{16}e^{-2k_1y}+k_{17}e^{-2k_2y}+k_{18}e^{-2k_6y}+k_{19}e^{-k_{11}y}+k_{20}e^{-k_{13}y}+k_{21}e^{-k_{15}y}+k_{22}e^{-k_2y} \tag{3.4.23}$$

where, $k_{16}=\dfrac{-\Pr k_7}{4k_1^{\,2}-2\Pr k_1-F}, k_{17}=\dfrac{-\Pr k_8}{4k_2^{\,2}-2\Pr k_2-F}, k_{18}=\dfrac{-\Pr k_9}{4k_6^{\,2}-2\Pr k_6-F}$,

$k_{19}=\dfrac{-\Pr k_{10}}{k_{11}^{\,2}-\Pr k_{11}-F},\ k_{20}=\dfrac{-\Pr k_{12}}{k_{13}^{\,2}-\Pr k_{13}-F},\ k_{21}=\dfrac{-\Pr k_{14}}{k_{15}^{\,2}-\Pr k_{15}-F},$

$k_{22}=-k_{16}-k_{17}-k_{18}-k_{19}-k_{20}-k_{21}$

Again we want to find the solution of u_1

$(3.4.6)\Rightarrow u_1''+u_1'-Nu_1=-Gm\phi_1-Gr\theta_1$

$$\Rightarrow u_1''+u_1'-Nu_1=-Gr\left(k_{16}e^{-2k_1y}+k_{17}e^{-2k_2y}+k_{18}e^{-2k_6y}+k_{19}e^{-k_{11}y}+k_{20}e^{-k_{13}y}+k_{21}e^{-k_{15}y}+k_{22}e^{-k_2y}\right)$$

[by (3.4.15) and (3.4.23)] (3.4.24)

Let $u_1(y)=e^{my}$ (where $e^{my}\neq 0$) is a probable solution of (3.4.24)

$\therefore u_1'(y)=me^{my}$

and $u_1''(y)=m^2e^{my}$

The auxiliary equation (A.E.) of (3.4.24):

$m^2e^{my}+me^{my}-Ne^{my}=0$

$\Rightarrow\left(m^2+m-N\right)e^{my}=0$

$\therefore m^2+m-N=0$ since $e^{my}\neq 0$

$\Rightarrow m=\dfrac{1}{2}\left\{-1\pm\sqrt{1+4N}\right\}$

Complimentary function (C.F.) : $u_{1_c}=C_1e^{\frac{1}{2}\left\{-1+\sqrt{1+4N}\right\}y}+C_2e^{-\frac{1}{2}\left\{1+\sqrt{1+4N}\right\}y}$

Now particular integral (P.I.) :

$$u_{1_p}=\frac{1}{D^2+D-N}\left\{-Gr\left(k_{16}e^{-2k_1y}+k_{17}e^{-2k_2y}+k_{18}e^{-2k_6y}+k_{19}e^{-k_{11}y}+k_{20}e^{-k_{13}y}+k_{21}e^{-k_{15}y}+k_{22}e^{-k_2y}\right)\right\}$$

$$=-Gr\left(\frac{1}{D^2+D-N}k_{16}e^{-2k_1y}+\frac{1}{D^2+D-N}k_{17}e^{-2k_2y}+\frac{1}{D^2+D-N}k_{18}e^{-2k_6y}\right.$$

$$\left.+\frac{1}{D^2+D-N}k_{19}e^{-k_{11}y}+\frac{1}{D^2+D-N}k_{20}e^{-k_{13}y}\right.$$

$$+\frac{1}{D^2+D-N}k_{21}e^{-k_{15}y}+\frac{1}{D^2+D-N}k_{22}e^{-k_2 y}\Bigg)$$

$$=-Gr\left(\frac{k_{16}}{4k_1^{\ 2}-2k_1-N}e^{-2k_1 y}+\frac{k_{17}}{4k_2^{\ 2}-2k_2-N}e^{-2k_2 y}+\frac{k_{18}}{4k_6^{\ 2}-2k_6-N}e^{-2k_6 y}\right.$$

$$\left.+\frac{k_{19}}{k_{11}^{\ 2}-k_{11}-N}e^{-k_{11}y}+\frac{k_{20}}{k_{13}^{\ 2}-k_{13}-N}e^{-k_{13}y}+\frac{k_{21}}{k_{15}^{\ 2}-k_{15}-N}e^{-k_{15}y}+\frac{k_{22}}{k_2^{\ 2}-k_2-N}e^{-k_2 y}\right)$$

So the general solution is $u_1=u_{1_c}+u_{1_p}$

$$\Rightarrow u_1=C_1e^{\frac{1}{2}\left\{-1+\sqrt{1+4N}\right\}y}+C_2e^{-\frac{1}{2}\left\{1+\sqrt{1+4N}\right\}y}-Gr\left(\frac{k_{16}}{4k_1^{\ 2}-2k_1-N}e^{-2k_1 y}+\frac{k_{17}}{4k_2^{\ 2}-2k_2-N}e^{-2k_2 y}\right.$$

$$+\frac{k_{18}}{4k_6^{\ 2}-2k_6-N}e^{-2k_6 y}+\frac{k_{19}}{k_{11}^{\ 2}-k_{11}-N}e^{-k_{11}y}+\frac{k_{20}}{k_{13}^{\ 2}-k_{13}-N}e^{-k_{13}y}$$

$$\left.+\frac{k_{21}}{k_{15}^{\ 2}-k_{15}-N}e^{-k_{15}y}+\frac{k_{22}}{k_2^{\ 2}-k_2-N}e^{-k_2 y}\right) \qquad (3.4.25)$$

Now by the boundary conditions

$u_1=0$ on $y=0$

$u_1\to 0$ as $y\to\infty$

we get C_1=0

and $C_2=Gr\left(\frac{k_{16}}{4k_1^{\ 2}-2k_1-N}+\frac{k_{17}}{4k_2^{\ 2}-2k_2-N}+\frac{k_{18}}{4k_6^{\ 2}-2k_6-N}+\frac{k_{19}}{k_{11}^{\ 2}-k_{11}-N}\right.$

$$\left.+\frac{k_{20}}{k_{13}^{\ 2}-k_{13}-N}+\frac{k_{21}}{k_{15}^{\ 2}-k_{15}-N}+\frac{k_{22}}{k_2^{\ 2}-k_2-N}\right)$$

Thus (3.4.25)$\Rightarrow u_1=k_{23}e^{-2k_1 y}+k_{24}e^{-2k_2 y}+k_{25}e^{-2k_6 y}+k_{26}e^{-k_{11}y}+k_{27}e^{-k_{13}y}+k_{28}e^{-k_{15}y}+k_{29}e^{-k_2 y}+k_{30}e^{-k_6 y}$

(3.4.26)

where, $k_{23}=\frac{-G\,\mathrm{r}\,k_{16}}{4k_1^{\ 2}-2k_1-N}$, $k_{24}=\frac{-G\,\mathrm{r}\,k_{17}}{4k_2^{\ 2}-2k_2-N}$, $k_{25}=\frac{-G\,\mathrm{r}\,k_{18}}{4k_6^{\ 2}-2k_6-N}$,

$k_{26}=\frac{-G\,\mathrm{r}\,k_{19}}{k_{11}^{\ 2}-k_{11}-N}$, $k_{27}=\frac{-G\,\mathrm{r}\,k_{20}}{k_{13}^{\ 2}-k_{13}-N}$, $k_{28}=\frac{-G\,\mathrm{r}\,k_{21}}{k_{15}^{\ 2}-k_{15}-N}$, $k_{29}=\frac{-G\,\mathrm{r}\,k_{22}}{k_2^{\ 2}-k_2-N}$,

$k_{30}=-k_{23}-k_{24}-k_{25}-k_{26}-k_{27}-k_{28}-k_{29}$

So $u = u_0 + Ecu_1$

$$= k_3 e^{-k_1 y} + k_4 e^{-k_2 y} + k_5 e^{-k_6 y} + Ec\left(k_{23} e^{-2k_1 y} + k_{24} e^{-2k_2 y} + k_{25} e^{-2k_6 y} + k_{26} e^{-k_{11} y}\right.$$

$$\left. + k_{27} e^{-k_{13} y} + k_{28} e^{-k_{15} y} + k_{29} e^{-k_2 y} + k_{30} e^{-k_6 y}\right) \quad \text{[by (3.4.20) and (3.4.26)]}$$

$$\therefore u = k_3 e^{-k_1 y} + \left(k_4 + Eck_{29}\right) e^{-k_2 y} + \left(k_5 + Eck_{30}\right) e^{-k_6 y} + Ec\left(k_{23} e^{-2k_1 y} + k_{24} e^{-2k_2 y} + k_{25} e^{-2k_6 y}\right.$$

$$\left. + k_{26} e^{-k_{11} y} + k_{27} e^{-k_{13} y} + k_{28} e^{-k_{15} y}\right) \tag{3.4.27}$$

Again $\theta = \theta_0 + Ec\theta_1$

$$= e^{-k_2 y} + Ec\left(k_{16} e^{-2k_1 y} + k_{17} e^{-2k_2 y} + k_{18} e^{-2k_6 y} + k_{19} e^{-k_{11} y} + k_{20} e^{-k_{13} y} + k_{21} e^{-k_{15} y} + k_{22} e^{-k_2 y}\right)$$

[by (3.4.17) and (3.4.23)]

$$\therefore \theta = \left(1 + Eck_{22}\right) e^{-k_2 y} + Ec\left(k_{16} e^{-2k_1 y} + k_{17} e^{-2k_2 y} + k_{18} e^{-2k_6 y} + k_{19} e^{-k_{11} y} + k_{20} e^{-k_{13} y} + k_{21} e^{-k_{15} y}\right)$$

(3.4.28)

And $\phi = \phi_0 + Ec\phi_1$

$$\therefore \phi = e^{-k_1 y} \qquad \text{[by (3.4.13) and (3.4.15)]} \tag{3.4.29}$$

The non-dimensional skin friction at the surface is $\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$

Now we have from (3.4.27)

$$u = k_3 e^{-k_1 y} + k_4 e^{-k_2 y} + k_5 e^{-k_6 y} + Ec\left(k_{23} e^{-2k_1 y} + k_{24} e^{-2k_2 y} + k_{25} e^{-2k_6 y} + k_{26} e^{-k_{11} y}\right.$$

$$\left. + k_{27} e^{-k_{13} y} + k_{28} e^{-k_{15} y} + k_{29} e^{-k_2 y} + k_{30} e^{-k_6 y}\right)$$

$$\therefore \frac{\partial u}{\partial y} = -k_1 k_3 e^{-k_1 y} - k_2 k_4 e^{-k_2 y} - k_5 k_6 e^{-k_6 y} - Ec\left(2k_1 k_{23} e^{-2k_1 y} + 2k_2 k_{24} e^{-2k_2 y} + 2k_6 k_{25} e^{-2k_6 y}\right.$$

$$\left. + k_{11} k_{26} e^{-k_{11} y} + k_{13} k_{27} e^{-k_{13} y} + k_{15} k_{28} e^{-k_{15} y} + k_2 k_{29} e^{-k_2 y} + k_6 k_{30} e^{-k_6 y}\right)$$

$$\therefore \tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -k_1 k_3 - k_2 k_4 - k_5 k_6 - Ec\left(2k_1 k_{23} + 2k_2 k_{24} + 2k_6 k_{25} + k_{11} k_{26} + k_{13} k_{27}\right.$$

$$\left. + k_{15} k_{28} + k_2 k_{29} + k_6 k_{30}\right)$$

The rate of heat transfer in terms of the Nusselt number is $Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$

Here we get from (3.4.28)

$$\theta = e^{-k_2 y} + Ec\left(k_{16}e^{-2k_1 y} + k_{17}e^{-2k_2 y} + k_{18}e^{-2k_6 y} + k_{19}e^{-k_{11} y} + k_{20}e^{-k_{13} y} + k_{21}e^{-k_{15} y} + k_{22}e^{-k_2 y}\right)$$

$$\Rightarrow \frac{\partial\theta}{\partial y} = -k_2 e^{-k_2 y} - Ec\left(2k_1k_{16}e^{-2k_1 y} + 2k_2k_{17}e^{-2k_2 y} + 2k_6k_{18}e^{-2k_6 y} + k_{11}k_{19}e^{-k_{11} y} + k_{13}k_{20}e^{-k_{13} y}\right.$$
$$\left. + k_{15}k_{21}e^{-k_{15} y} + k_2k_{22}e^{-k_2 y}\right)$$

$$\therefore\ Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = k_2 + Ec\left(2k_1k_{16} + 2k_2k_{17} + 2k_6k_{18} + k_{11}k_{19} + k_{13}k_{20} + k_{15}k_{21} + k_2k_{22}\right)$$

The Sherwood number which is in non-dimensional form is $S_h = -\left(\frac{\partial\phi}{\partial y}\right)_{y=0}$

Now from equation (3.4.29), $\phi = e^{-k_1 y}$

$$\Rightarrow \frac{\partial\phi}{\partial y} = -k_1 e^{-k_1 y}$$

$$\therefore S_h = -\left(\frac{\partial\phi}{\partial y}\right)_{y=0} = k_1$$

Chapter 4

Results and Discussion

To observe the physical situation of the problem of our study the effects of Prandtl number (*Pr*), Schmidt number (*Sc*), magnetic parameter (*M*), Grashof number (*Gr*), modified Grashof number (*Gm*), Eckert number (*Ec*), Hall parameter (*m*), chemical reaction parameter (*Kc*), permeability parameter (*k*), radiation parameter (*F*) on velocity field, temperature field, concentration field, skin-friction, the rate of heat transfer in terms of Nusselt number (*Nu*) and the rate of mass transfer in terms of Sherwood number (*Sh*) are studied taking different numerical values. To see the effects of these parameters, the values of Schmidt numbers (*Sc*) are chosen for hydrogen (*Sc*=0.22), water-vapor (*Sc*=0.60), ammonia (*Sc*=0.78) at 25ºC and one atmosphere pressure. The values of Prandtl numbers (*Pr*) are chosen for sodium (*Pr*=0.01), air (*Pr*=0.71) and water (*Pr*=7.0). We also choose the Grashof numbers (*Gr*) for heat transfer are *Gr*=5.0, 6.0, 10.0 and modified Grashof numbers for mass transfer are *Gm*=2.0, 3.0, 4.0. The values of magnetic parameter are given *M*=1.0, 3.0, 5.0 arbitrarily.

The velocity profiles *u* for different values of the above parameters are illustrated in Figure 4.1.1 to Figure 4.1.10, the temperature profiles for different values of the parameters are described in Figure 4.2.1 to Figure 4.2.3 and the concentration profiles for different values of the above parameters are expressed in Figure 4.3.1 to Figure 4.3.2. We compare velocity profiles in presence of magnetic field and without magnetic field in the Figure 4.4.1 to Figure 4.4.18. Also the numerical values of Skin-Friction ( ), the rate of heat transfer (*Nu*) and the rate of mass transfer (*Sh*) are shown in the Table 4.5.1 to Table 4.5.3.

4.1 Velocity Profiles

The Figure 4.1.1 to Figure 4.1.10 depict the velocity distribution u for different values of magnetic parameter (M), Hall parameter (m), chemical reaction parameter (Kc), permeability parameter (k), radiation parameter (F), Grashof number (Gr), modified Grashof number (Gm), Schmidt number (Sc), Prandtl number (Pr) and Eckert number (Ec) respectively.

In the Figure 4.1.1 it is observed that the velocity decreases with the increase of magnetic parameter (M). Physically this is true as the magnetic force retards the flow, velocity decreases. In this figure the dotted line represents for M=5.0, solid line for M=3.0 and dashing line for M=1.0.

Figure 4.1.2 shows the velocity distributions for different values of Hall parameter (m). After analyzing the figure it is noticed that the velocity increases with the increase of Hall parameter (m). In this figure the dashing line denotes for m=1.0, solid line for m=2.0 and dotted line for m=3.0. For our interest, we calculate the increasing rate of the velocity at the corresponding point y=5 of the curve from m=1.0 to m=2.0 is 8% and from m=2.0 to m=3.0 is 4%.

It is described in the Figure 4.1.3 that the velocity distributions for different values of chemical reaction parameter (Kc). In this figure we observe the velocity decreases with the increase of chemical reaction parameter (Kc). In this figure the dashing line indicates for Kc=−0.04, solid line for Kc=0.0 and dotted line for Kc=0.04.

We see in Figure 4.1.4 that the velocity increases with the increase of permeability parameter (k). In this figure the pick points of the curves for k=0.1, k=0.5 and k=1.0 are obtained at y=0.55, y=0.8 and y=1.0 respectively and after arriving the pick point it decreases smoothly. Here the dashing line denotes for k=0.1, solid line for k=0.5 and dotted line for k=1.0. For our interest, we have calculated the increasing rate of the velocity at the corresponding point y=4 of the curve from k=0.1 to k=0.5 is 77.5% and from k=0.5 to k=1.0 is 54%.

Figure 4.1.5 indicates that the velocity decreases with the increase of radiation parameter (F). The dashing line denotes for F=0.5, solid line for F=5.0 and dotted line for F=10.0.

From Figure 4.1.6 the velocity increases with the increase of Grashof number (Gr). The dashing line represents for Gr=5.0, solid line for Gr=6.0 and dotted line for Gr=10.0. At y=0 the velocity profiles are zero.

In the Figure 4.1.7 it is observed that the velocity increases with the increase of modified Grashof number (Gm). The dashing line indicates for Gm=2.0, solid line for Gm=3.0 and dotted line for Gm=4.0.

It is clear in Figure 4.1.8 that the velocity decreases with the increase of Schmidt number (Sc). In this figure the dotted line represents for Sc=0.78, solid line for Sc=0.60 and dashing line for Sc=0.22. For our interest, we calculate the decreasing rate of the velocity at the corresponding point y=4 of the curve from Sc=0.22 to Sc=0.60 is 81.58% and from Sc=0.60 to Sc=0.78 is 27.78%.

Also Figure 4.1.9 marks that the velocity decreases with the increase of Prandtl number (Pr). Physically it is true because the increase in the Prandtl number due to increasing the viscosity of the fluid which makes the fluid thick and hence decrease the velocity of fluid. In this figure the dashing line denotes for Pr=0.01, solid line for Pr=0.70 and dotted line for Pr=7.0.

The velocity distributions for different values of Eckert number (Ec) is shown in the Figure 4.1.10. In this figure we observe the velocity increases with the increase of Eckert number (Ec). In this figure the dashing line indicates for Ec=0.001, solid line for Ec=0.09 and dotted line for Ec=0.3.

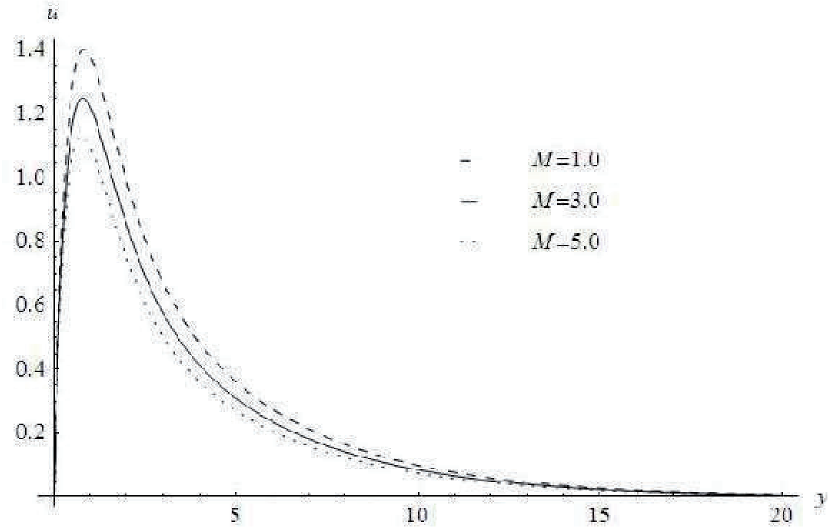

Figure 4.1.1 Variation of velocity profiles *u* for different values of magnetic parameter *M* where *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

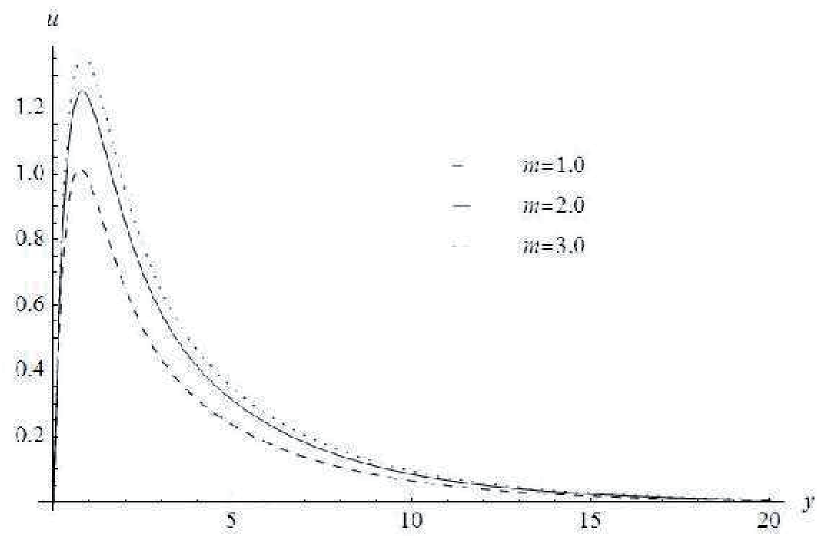

Figure 4.1.2 Variation of velocity profiles *u* for different values of hall parameter *m* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *k*=0.5 and *Ec*=0.01 against *y*.

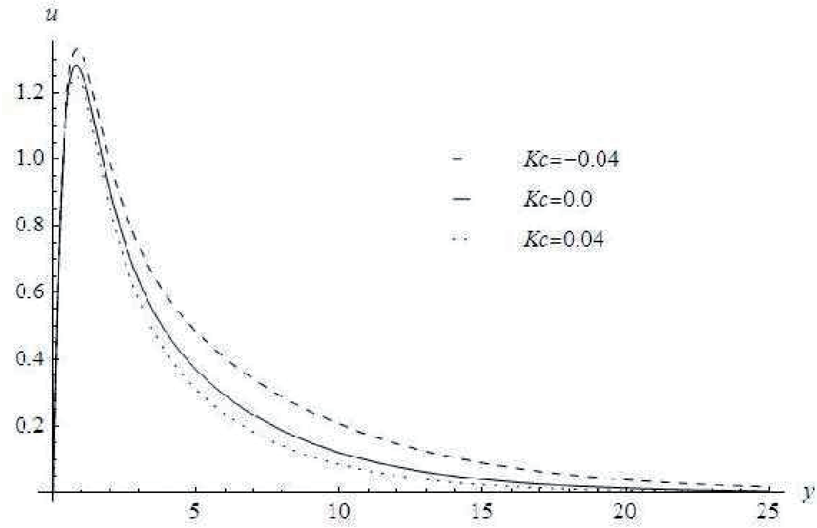

Figure 4.1.3 Variation of velocity profiles *u* for different values of chemical reaction parameter *Kc* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

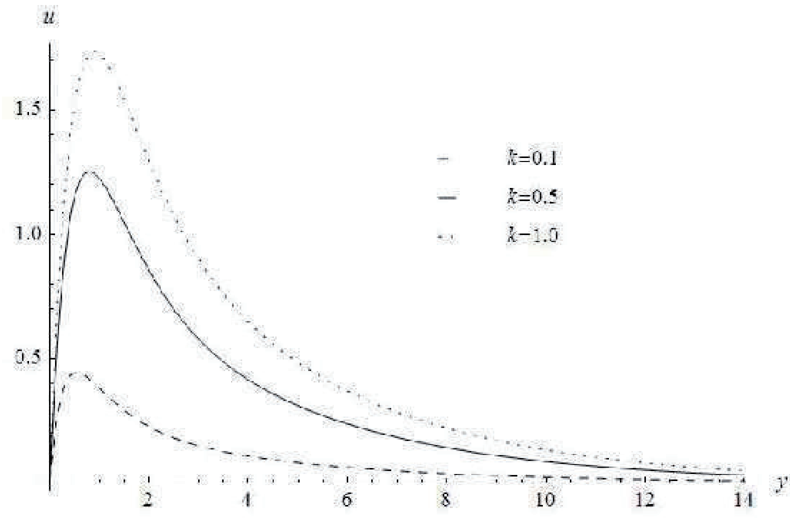

Figure 4.1.4 Variation of velocity profiles *u* for different values of permeability parameter *k* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2 and *Ec*=0.01 against *y*.

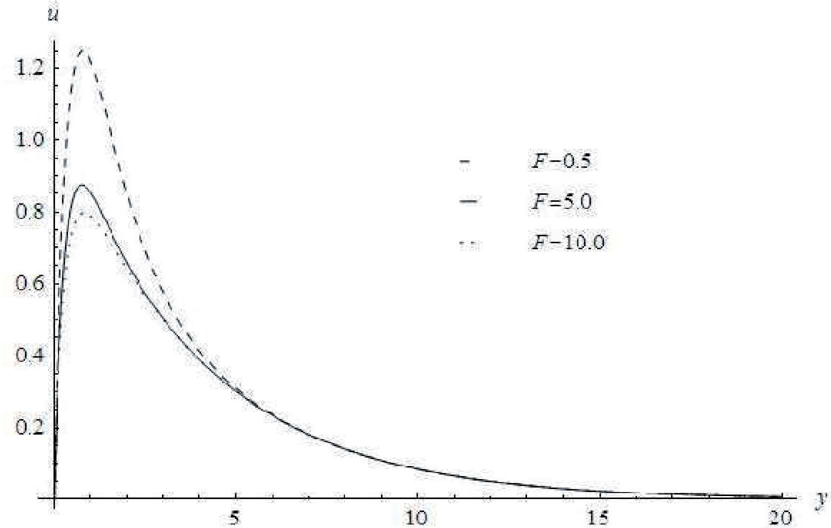

Figure 4.1.5 Variation of velocity profiles *u* for different values of radiation parameter *F* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

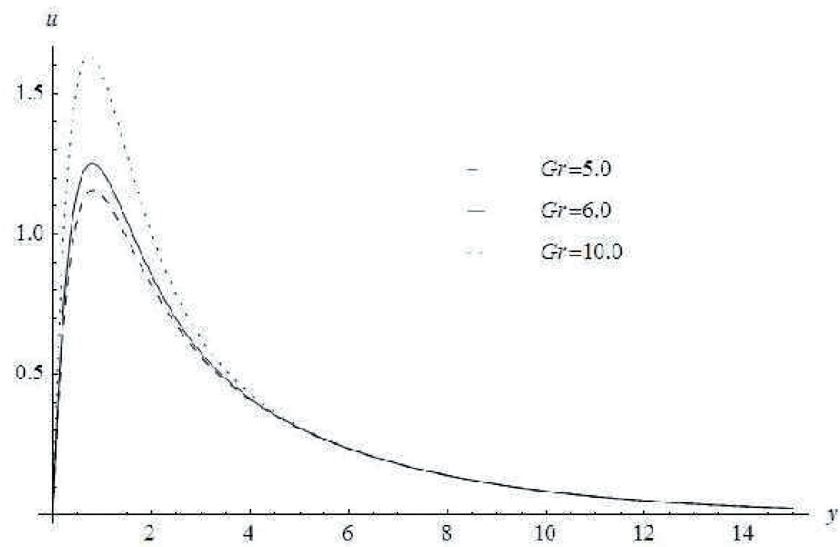

Figure 4.1.6 Variation of velocity profiles *u* for different values of Grashof number *Gr* where *M*=3.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

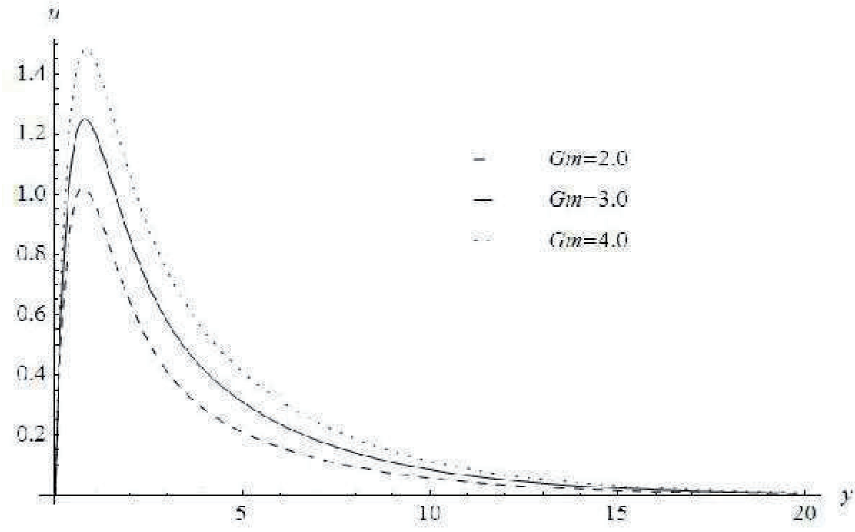

Figure 4.1.7 Variation of velocity profiles *u* for different values of modified Grashof number *Gm* where *M*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

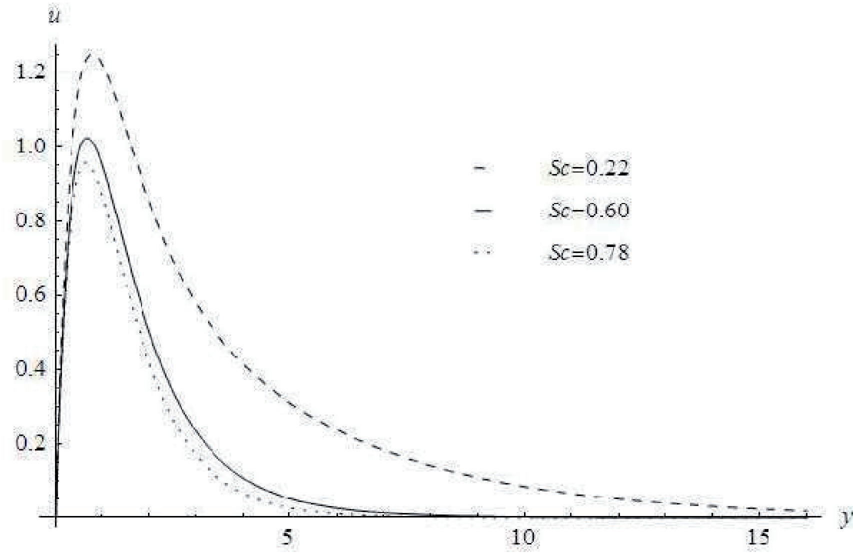

Figure 4.1.8 Variation of velocity profiles *u* for different values of Schmidt number *Sc* where *M*=3.0, *Gm* =3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

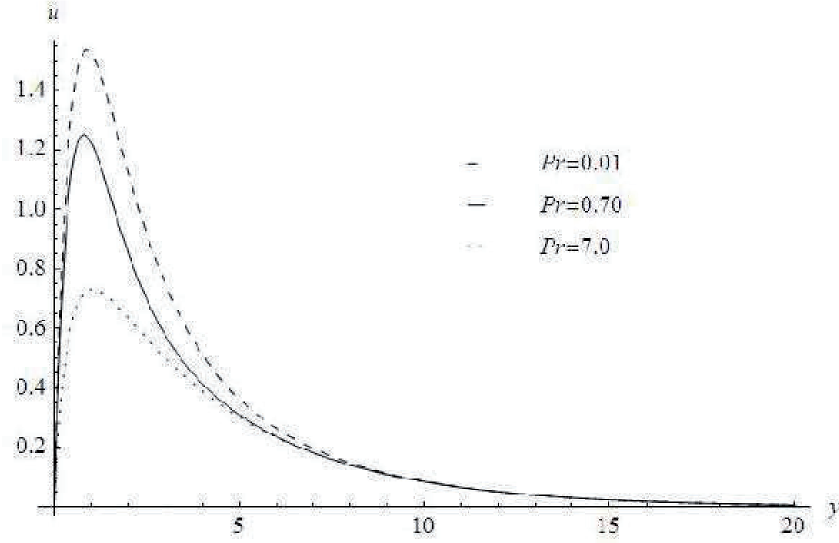

Figure 4.1.9 Variation of velocity profiles *u* for different values of Prandtl number *Pr* where *M*=3.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

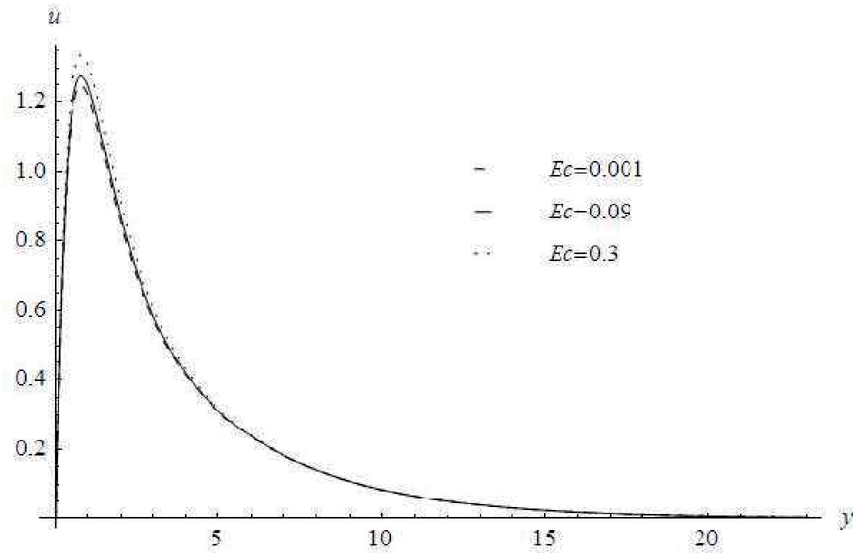

Figure 4.1.10 Variation of velocity profiles *u* for different values of Eckert number *Ec* where *M*=3.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2, *k*=0.5 and *Pr*=0.71 against *y*.

4.2 Temperature Distributions

The temperature distributions for different values of radiation parameter (F), Prandtl number (Pr) and Eckert number (Ec) are shown in the Figure 4.2.1 to Figure 4.2.3 respectively.

From Figure 4.2.1 it is clear that the temperature decreases with the increase of radiation parameter (F). The dashing line denotes for F=0.5, solid line for F=5.0 and dotted line for F=10.0. At y=0 the temperature profiles attain the maximum value 1.0 and then decrease smoothly and attain to zero with the increase of y.

It is observed in the Figure 4.2.2 that the temperature distributions for different values of Prandtl number (Pr). In this figure it is noticed that the temperature decreases with the increase of Prandtl number (Pr). The dashing line denotes for Pr=0.01, solid line for Pr=0.71 and dotted line for Pr=7.0. The temperature profiles attain the maximum value 1.0 at y=0 and then gradually attain nearly to zero for large values of y.

Figure 4.2.3 shows the temperature distributions for different values of Eckert number (Ec). After analysing the figure it is noticed that the temperature increases with the increase of Eckert number (Ec). In this figure the dashing line indicates for Ec=0.001, solid line for Ec=0.09 and dotted line for Ec=0.3.

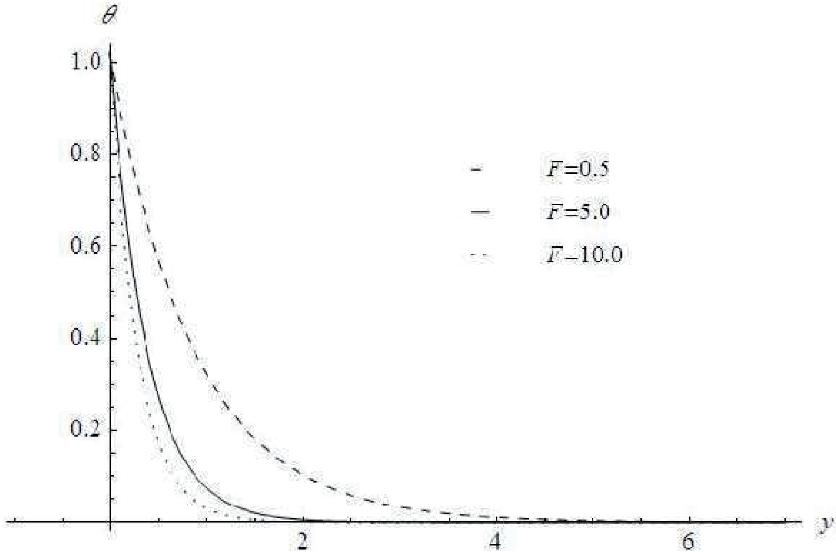

Figure 4.2.1 Variation of temperature profiles for different values of radiation parameter *F* where *M*=3.0, *Gr*=6.0, *Gm* =3.0, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

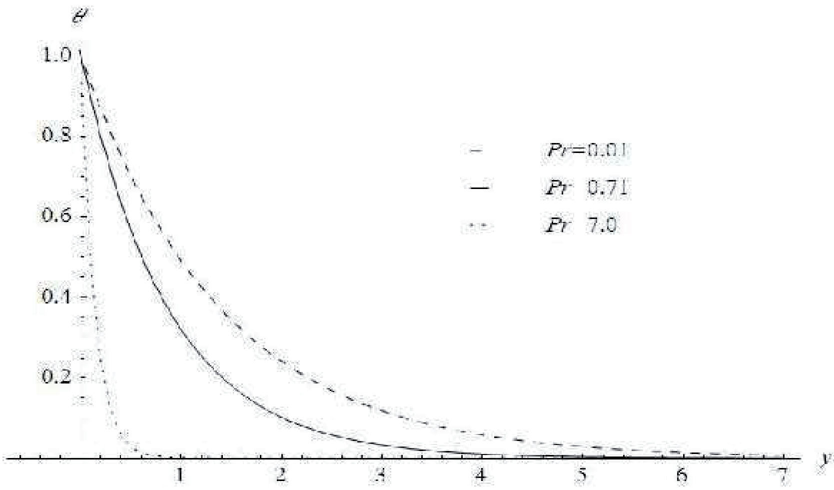

Figure 4.2.2 Variation of temperature profiles for different values of Prandtl number *Pr* where *M*=3.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

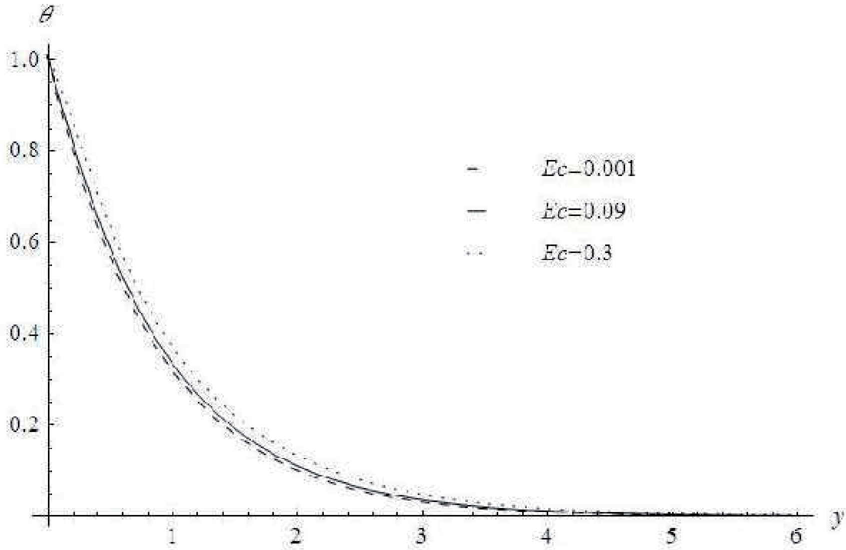

Figure 4.2.3 Variation of temperature profiles *u* for different values of Eckert number *Ec* where *M*=3.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2, *k*=0.5 and *Pr*=0.71 against *y*.

4.3 Concentration Distributions

From the Figure 4.3.1 and Figure 4.3.2 it is observed that the concentration distributions for different values of chemical reaction parameter (*Kc*) and Schmidt number (*Sc*).

In the Figure 4.3.1 it depicts that the concentration decreases with the increase of reaction parameter (*Kc*). The dashing line indicates for $Kc=-0.04$, solid line for $Kc=0.0$ and dotted line for $Kc=0.04$. In this figure maximum value of concentration profiles for $y=0$ is 1.0 the concentration profiles decrease smoothly and attain to zero for large value of *y*.

Figure 4.3.2 marks that the concentration decreases with the increase of Schmidt number (*Sc*). The dashing line indicates for $Sc=0.22$, solid line for $Sc=0.60$ and dotted line for $Sc=0.78$. We get the maximum value of concentration profiles for $y=0$ the concentration profiles gradually attain to zero with the increase of *y*.

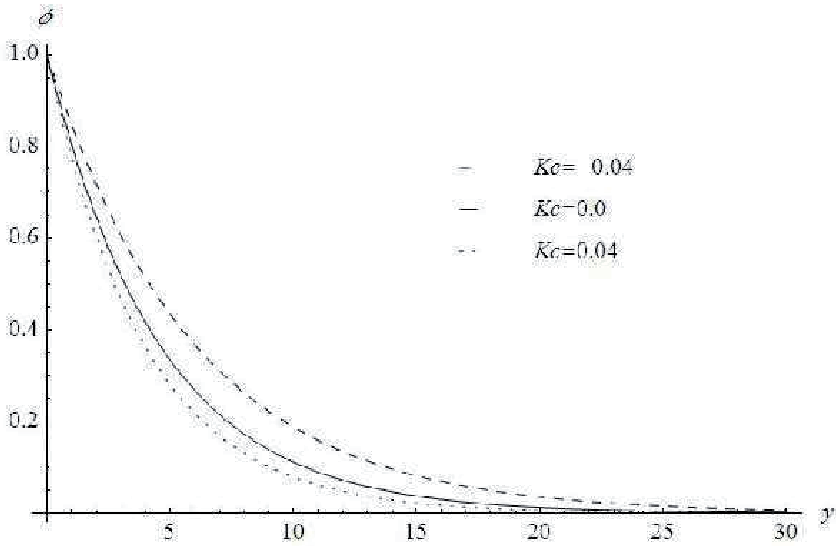

Figure 4.3.1 Variation of concentration profiles for different values of chemical reaction parameter *Kc* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Sc*=0.22, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

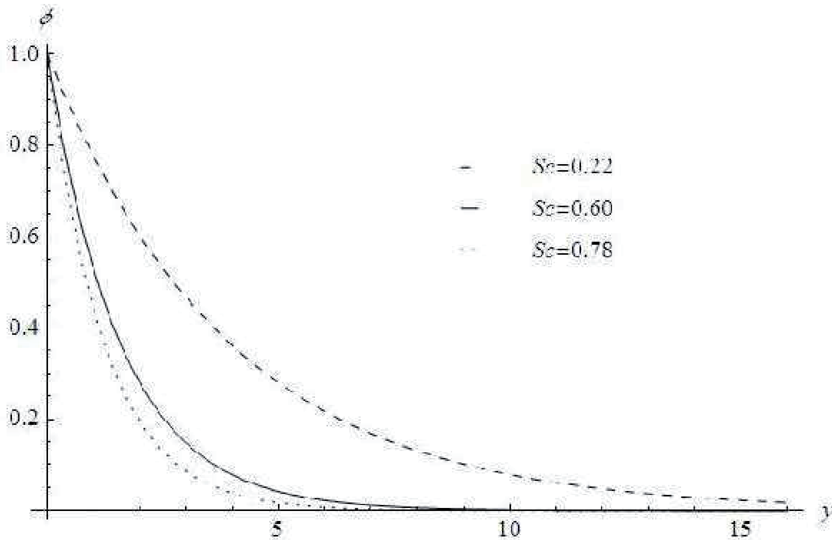

Figure 4.3.2 Variation of concentration profiles for different values of Schmidt number *Sc* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Pr*=0.71, *m*=2, *k*=0.5 and *Ec*=0.01 against *y*.

4.4 Comparison

Figure 4.4.1 shows the velocity profiles for magnetic field (*M*=3.0) with the different values of Hall parameter (*m*). On the other hand Figure 4.4.2 represents the graphs without magnetic field (*M*=0) for different values of hall parameter (*m*). In these figures we see that the graphs of velocity profiles coincides for *M*=0, that is there is no effect of Hall parameter (*m*) for magnetic field *M*=0.

Figure 4.4.3, Figure 4.4.5, Figure 4.4.7, Figure 4.4.9, Figure 4.4.11, Figure 4.4.13, Figure 4.4.15 and Figure 4.4.17 depict the velocity profiles for magnetic field (*M*=3.0) with the different values of chemical reaction parameter *Kc*, permeability parameter *k*, radiation parameter *F*, Grashof number *Gr*, modified Grashof number *Gm*, Schmidt number *Sc*, Prandtl number *Pr* and Eckert number (*Ec*) respectively. Whereas Figure 4.4.4, Figure 4.4.6, Figure 4.4.8, Figure 4.4.10, Figure 4.4.12, Figure 4.4.14, Figure 4.4.16 and Figure 4.4.18 represents the graphs without magnetic field (*M*=0) for different values of *Kc*, *k*, *F*, *Gr*, *Gm*, *Sc Pr* and *Ec* respectively. From the comparison of the Figure 4.4.3 and Figure 4.4.4; Figure 4.4.5 and Figure 4.4.6; Figure 4.4.7 and Figure 4.4.8; Figure 4.4.9 and Figure 4.4.10; Figure 4.4.11 and Figure 4.4.12; Figure 4.4.13 and Figure 4.4.14; Figure 4.4.15 and Figure 4.4.16; Figure 4.4.17 and Figure 4.4.18 it is clear that the velocity in magnetic field is less than without magnetic field.

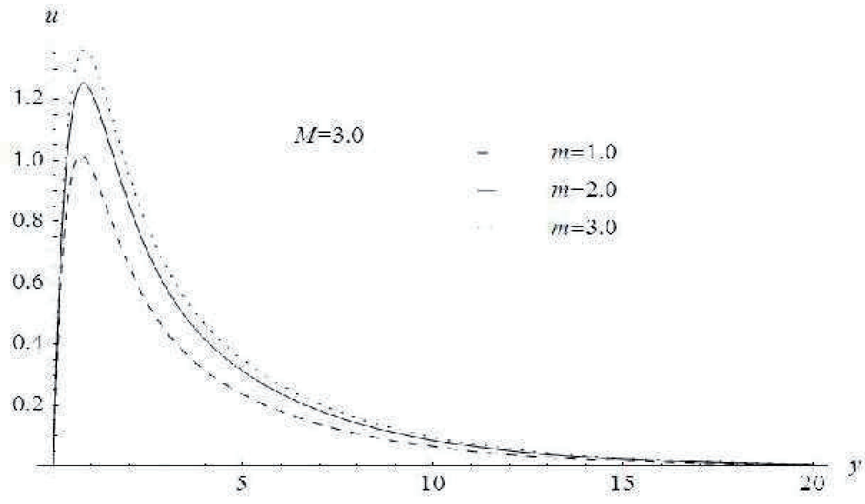

Figure 4.4.1 Variation of velocity profiles *u* for different values of hall parameter *m* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *k*=0.5 and *Ec*=0.01 against *y*.

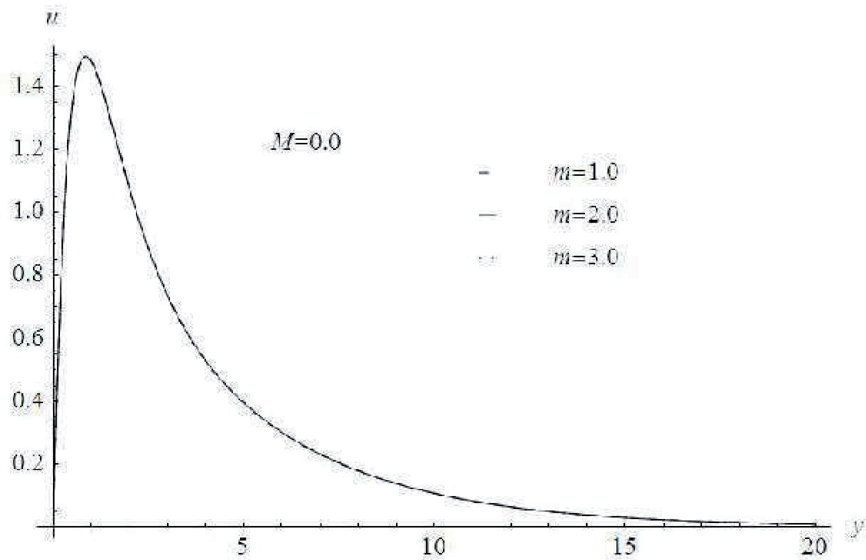

Figure 4.4.2 Variation of velocity profiles *u* for different values of hall parameter *m* where *M*=0.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *k*=0.5 and *Ec*=0.01 against *y*.

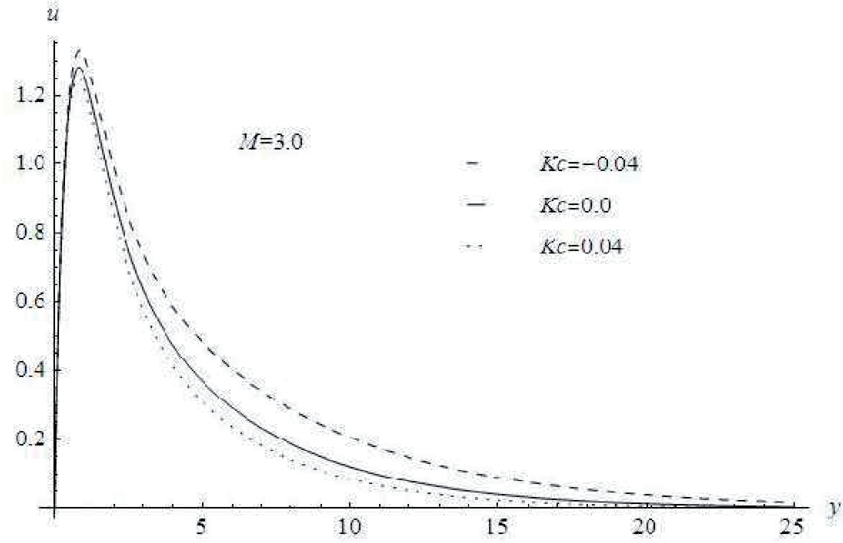

Figure 4.4.3 Variation of velocity profiles u for different values of chemical reaction parameter Kc where M=3.0, Gr=6.0, Gm=3.0, F=0.5, Kc=0.04, Sc=0.22, Pr=0.71, m=2.0, k=0.5 and Ec=0.01 against y.

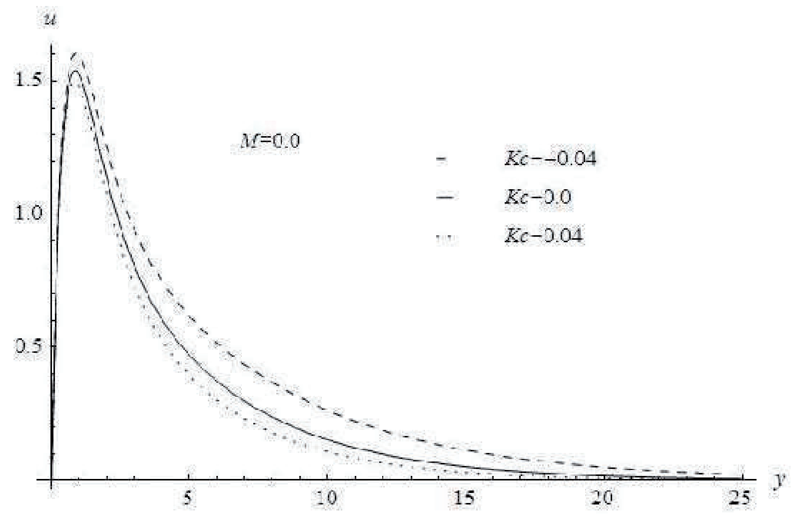

Figure 4.4.4 Variation of velocity profiles u for different values of chemical reaction parameter Kc where M=0.0, Gr=6.0, Gm=3.0, F=0.5, Kc=0.04, Sc=0.22, Pr=0.71, m=2.0, k=0.5 and Ec=0.01 against y.

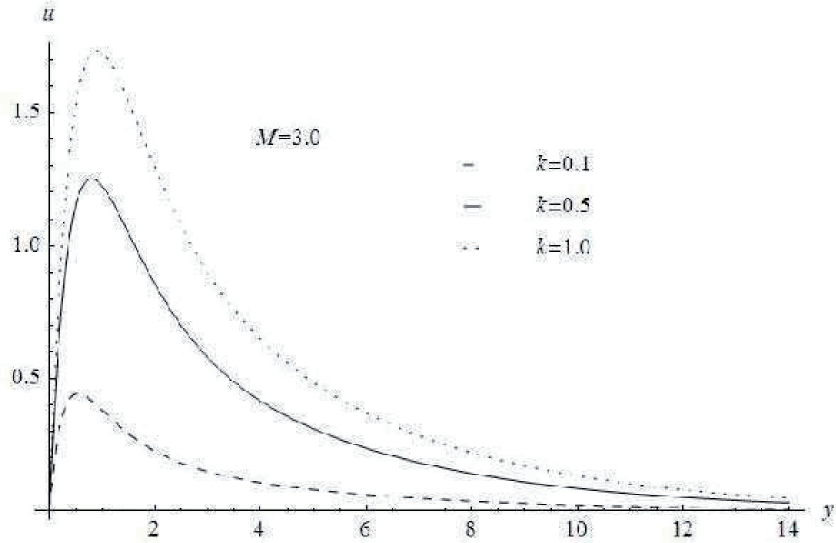

Figure 4.4.5 Variation of velocity profiles *u* for different values of permeability parameter *k* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0 and *Ec*=0.01 against *y*.

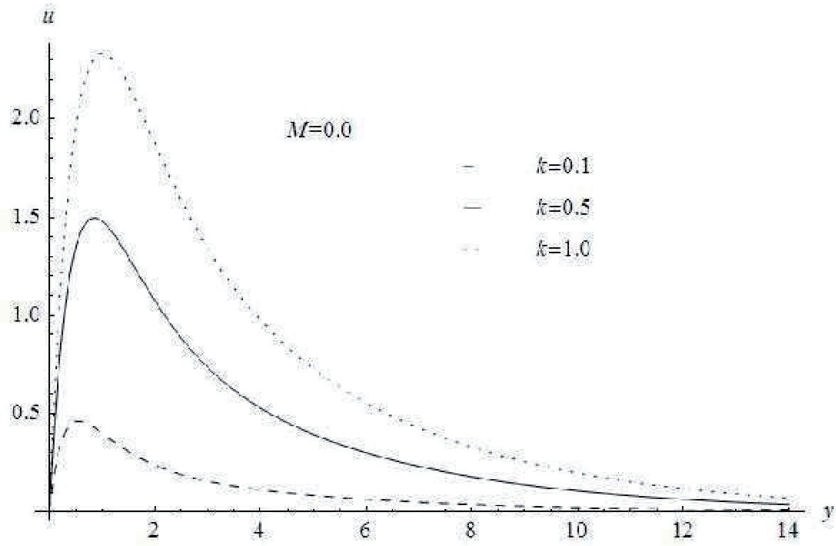

Figure 4.4.6 Variation of velocity profiles *u* for different values of permeability parameter *k* where *M*=0.0, *Gr*=6.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0 and *Ec*=0.01 against *y*.

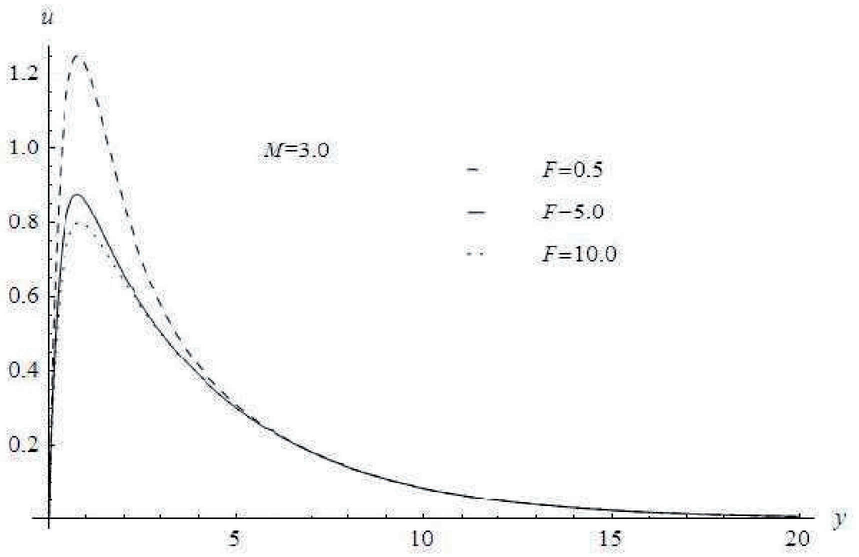

Figure 4.4.7 Variation of velocity profiles *u* for different values of radiation parameter *F* where *M*=3.0, *Gr*=6.0, *Gm*=3.0, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

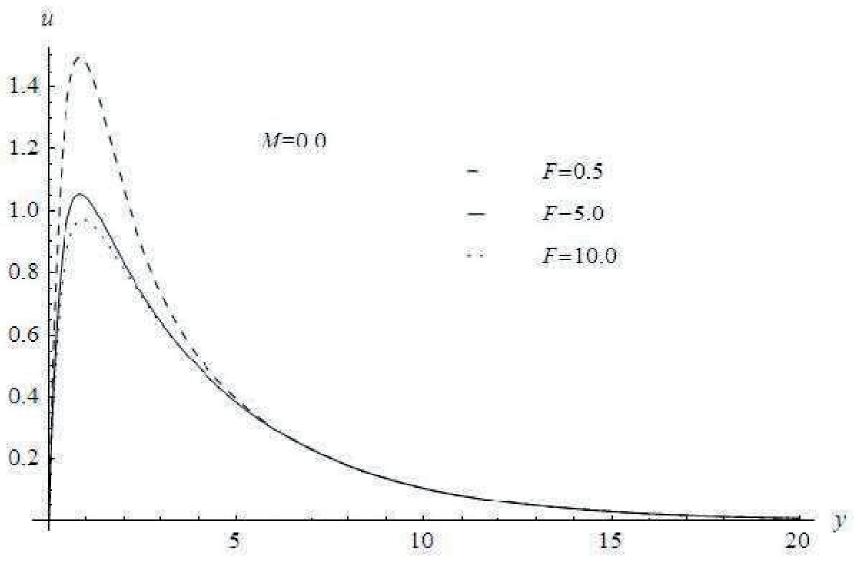

Figure 4.4.8 Variation of velocity profiles *u* for different values of radiation parameter *F* where *M*=0.0, *Gr*=6.0, *Gm*=3.0, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

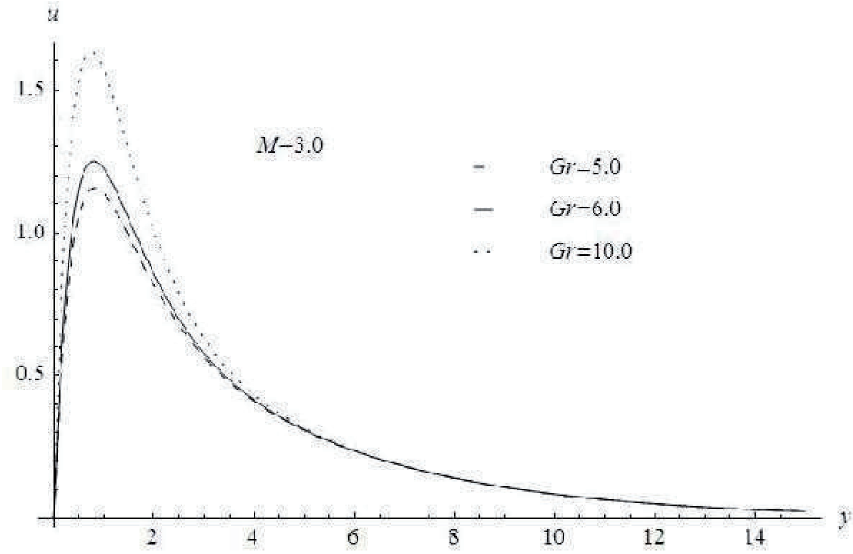

Figure 4.4.9 Variation of velocity profiles *u* for different values of Grashof number *Gr* where *M*=3.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

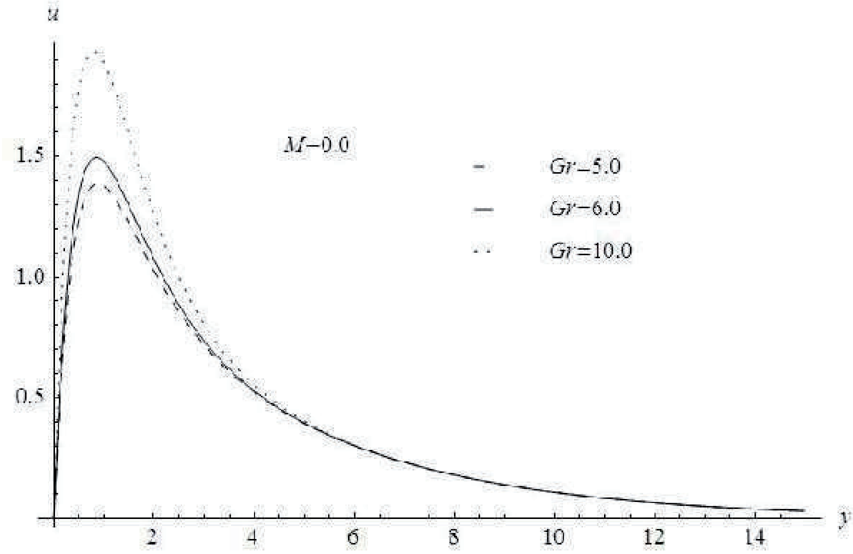

Figure 4.4.10 Variation of velocity profiles *u* for different values of Grashof number *Gr* where *M*=0.0, *Gm*=3.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

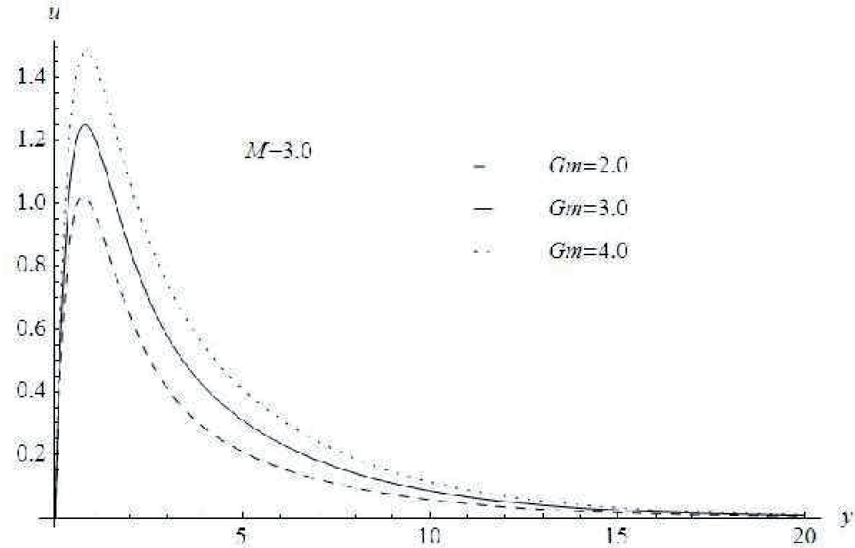

Figure 4.4.11 Variation of velocity profiles *u* for different values of modified Grashof number *Gm* where *M*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

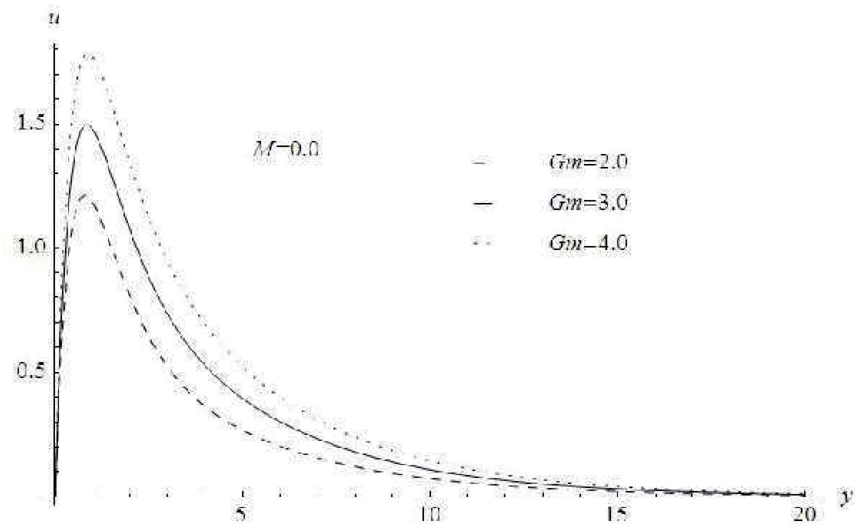

Figure 4.4.12 Variation of velocity profiles *u* for different values of modified Grashof number *Gm* where *M*=0.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *Pr*=0.71, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

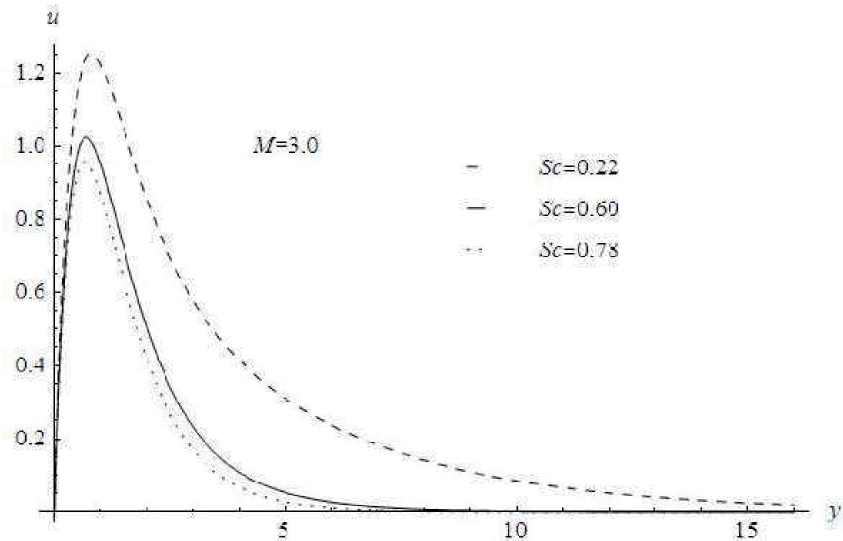

Figure 4.4.13 Variation of velocity profiles u for different values of Schmidt number Sc where M=3.0, Gm =3.0, Gr=6.0, F=0.5, Kc=0.04, Pr=0.71, m=2.0, k=0.5 and Ec=0.01 against y.

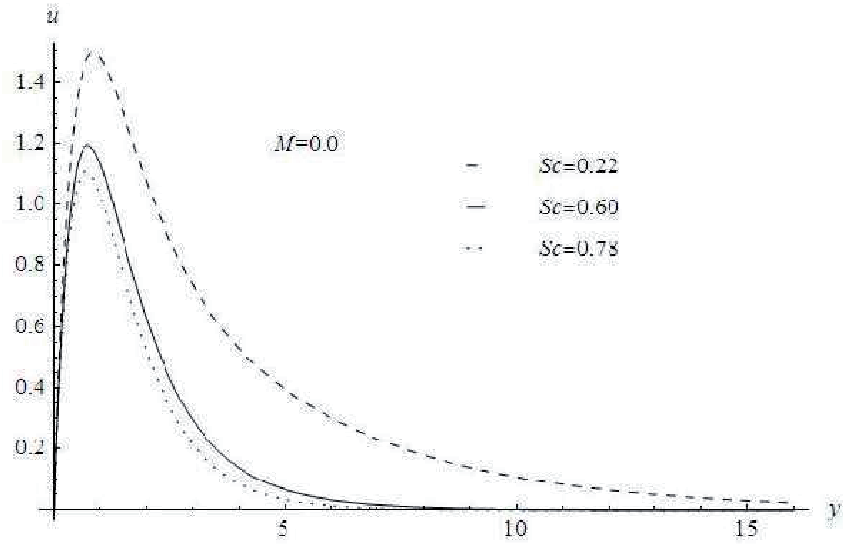

Figure 4.4.14 Variation of velocity profiles u for different values of Schmidt number Sc where M=0.0, Gm =3.0, Gr=6.0, F=0.5, Kc=0.04, Pr=0.71, m=2.0, k=0.5 and Ec=0.01 against y.

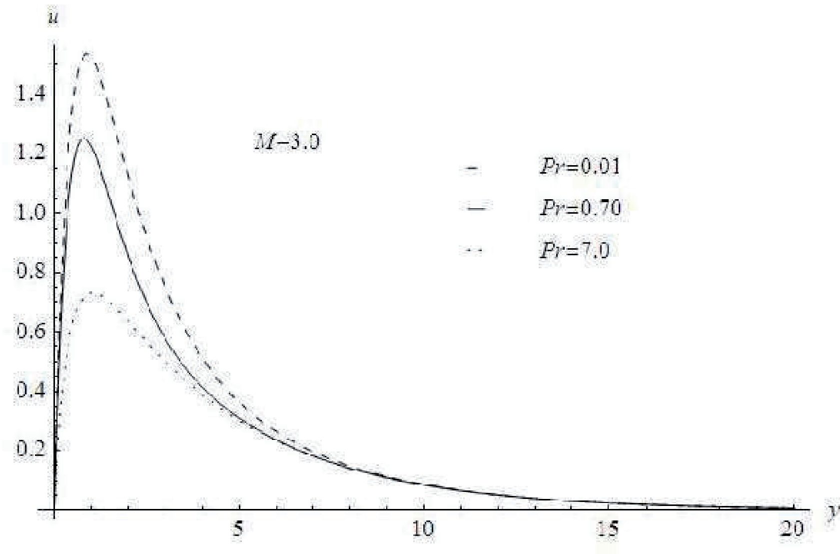

Figure 4.4.15 Variation of velocity profiles *u* for different values of Prandtl number *Pr* where *M*=3.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

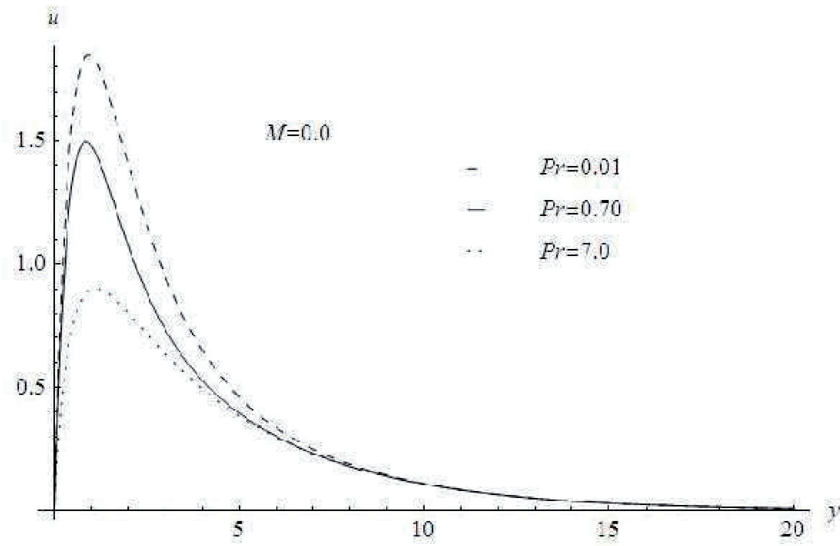

Figure 4.4.16 Variation of velocity profiles *u* for different values of Prandtl number *Pr* where *M*=0.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2.0, *k*=0.5 and *Ec*=0.01 against *y*.

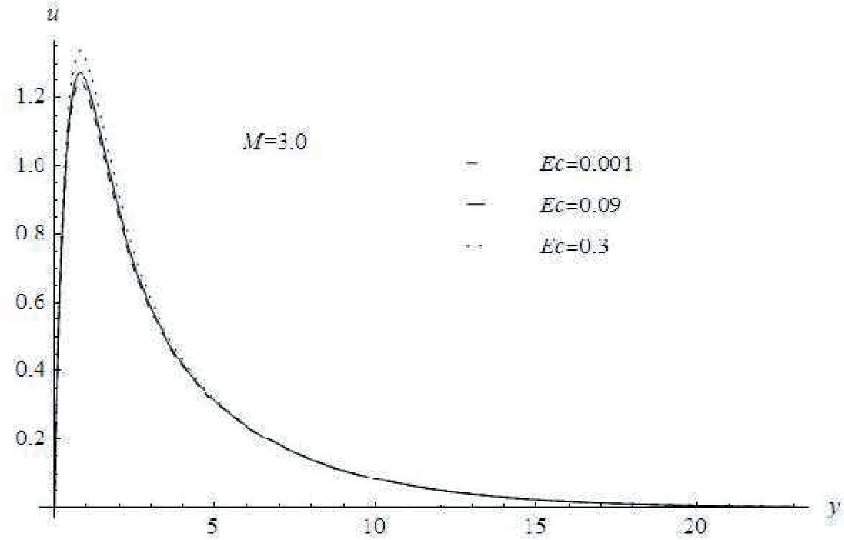

Figure 4.4.17 Variation of velocity profiles *u* for different values of Eckert number *Ec* where *M*=3.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2.0, *k*=0.5 and *Pr*=0.71 against *y*.

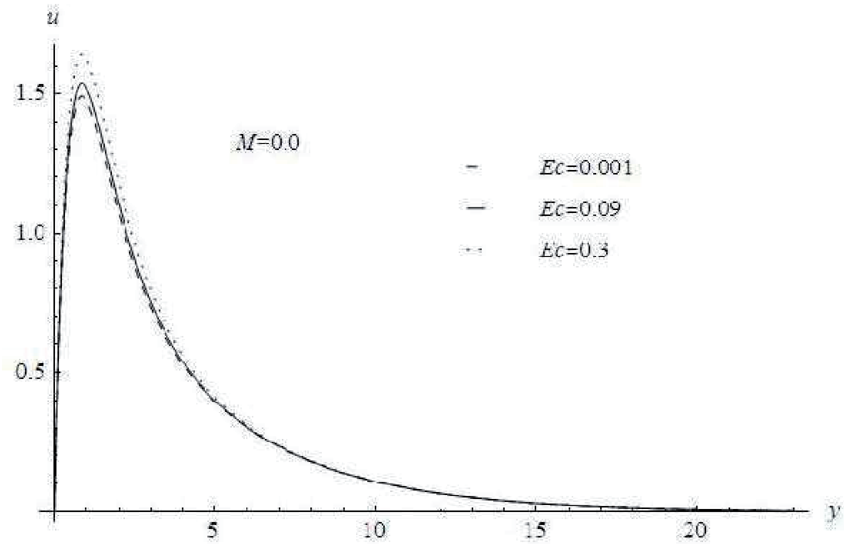

Figure 4.4.18 Variation of velocity profiles *u* for different values of Eckert number *Ec* where *M*=0.0, *Gm*=3.0, *Gr*=6.0, *F*=0.5, *Kc*=0.04, *Sc*=0.22, *m*=2.0, *k*=0.5 and *Pr*=0.71 against *y*.

4.5 Skin-friction ( ), Nusselt number (*Nu*) and Sherwood number(*Sh*)

A variation in skin friction ( ) is shown in Table 4.5.1. From this table it is observed that skin friction ( ) increases with the increase in Hall parameter (*m*) and permeability parameter (*k*) whereas it decreases with the increase of Magnetic parameter (*M*), radiation parameter (*F*) and chemical reaction parameter (*Kc*).

A variation in the heat transfer rate expressed in terms of the Nusselt number is presented Table 4.5.2. From this table it is noticed that Nusselt number (*Nu*) increases with the increase of magnetic parameter (*M*), radiation parameter (*F*) and chemical reaction parameter (*Kc*) and Nusselt number (*Nu*) decreases with the increase of Hall parameter (*m*) and permeability parameter (*k*).

Similarly a variation in the Sherwood number is shown in Table 4.5.3. It is clear from this table that Sherwood number (*Sh*) increases with the increase of Schmidt number (*Sc*) and chemical reaction parameter (*Kc*).

Table 4.5.1 Numerical values of Skin-Friction ( )

Sl. No.	*M*	*m*	*F*	*Kc*	*k*	
1	1.0	2.0	0.5	0.04	0.5	4.99799
2	3.0	2.0	0.5	0.04	0.5	4.65836
3	5.0	2.0	0.5	0.04	0.5	4.38312
4	3.0	1.0	0.5	0.04	0.5	4.10228
5	3.0	2.0	0.5	0.04	0.5	4.65836
6	3.0	3.0	0.5	0.04	0.5	4.90566
7	3.0	2.0	0.5	0.04	0.5	4.65836
8	3.0	2.0	5.0	0.04	0.5	3.65714
9	3.0	2.0	10.0	0.04	0.5	3.35018
10	3.0	2.0	0.5	−0.04	0.5	4.79238
11	3.0	2.0	0.5	0.0	0.5	4.70956
12	3.0	2.0	0.5	0.04	0.5	4.65836
13	3.0	2.0	0.5	0.04	0.1	2.50745
14	3.0	2.0	0.5	0.04	0.5	4.65836
15	3.0	2.0	0.5	0.04	1.0	5.70216

Table 4.5.2 Numerical values of the rate of Heat Transfer (*Nu*)

Sl. No.	***M***	***m***	***F***	***Kc***	***k***	***Nu***
1	1.0	2.0	0.5	0.04	0.5	1.11868
2	3.0	2.0	0.5	0.04	0.5	1.12324
3	5.0	2.0	0.5	0.04	0.5	1.12659
4	3.0	1.0	0.5	0.04	0.5	1.12970
5	3.0	2.0	0.5	0.04	0.5	1.12324
6	3.0	3.0	0.5	0.04	0.5	1.11996
7	3.0	2.0	0.5	0.04	0.5	1.12324
8	3.0	2.0	5.0	0.04	0.5	2.60881
9	3.0	2.0	10.0	0.04	0.5	3.52950
10	3.0	2.0	0.5	−0.04	0.5	1.12154
11	3.0	2.0	0.5	0.0	0.5	1.12260
12	3.0	2.0	0.5	0.04	0.5	1.12324
13	3.0	2.0	0.5	0.04	0.1	1.14178
14	3.0	2.0	0.5	0.04	0.5	1.12324
15	3.0	2.0	0.5	0.04	1.0	1.10770

Table 4.5.3 Numerical values of the rate of Mass Transfer (*Sh*)

Sl. No.	***Sc***	***Kc***	***Sh***
1	0.22	0.04	0.254568
2	0.60	0.04	0.637639
3	0.78	0.04	0.818135
4	0.22	−0.04	0.167446
5	0.22	0.0	0.220000
6	0.22	0.04	0.254568

Chapter 5

Conclusion

In the present research work, we have studied the effects of Hall current, chemical reaction and radiation on MHD free convection flow through a vertical plate embedded in porous medium. The results are given graphically to illustrate the variation of velocity, temperature and concentration with different parameters. Also the Nusselt number, Sherwood number and skin-friction are presented in tables. From the analysis of the study the following conclusions are made:

1. The velocity profiles increase with the increase of Hall parameter (m), permeability parameter (k), Grashof number (Gr), modified Grashof number (Gm). On the other hand, it decrease with the increase of Magnetic parameter (M), Chemical reaction parameter (Kc), Radiation parameter (F), Schmidt number (Sc), Prandtl number (Pr).
2. The temperature distributions decrease with the increase of Radiation parameter (F) and Prandtl number (Pr).
3. The concentration distributions decrease with the increase of Chemical reaction parameter (Kc) and Schmidt number (Sc).
4. From the comparison of velocity profiles in magnetic field and without magnetic field for different parameters we have concluded that the velocity without magnetic field is more than the velocity in magnetic field. Moreover there is a exception, without magnetic field the velocity graphs coincide for different values of Hall parameter (m).
5. The skin friction ( ) increases with the increase of Hall parameter (m) and permeability parameter (k) whereas it decreases with the increase of Magnetic parameter (M), radiation parameter (F) and chemical reaction parameter (Kc).

6. The heat transfer rate expressed in terms of the Nusselt number(Nu) increases with the increase of Magnetic parameter (M), Radiation parameter (F) and Chemical reaction parameter (Kc) and decreases with the increase of Hall parameter (m) and permeability parameter (k).
7. The mass transfer rate expressed in terms of Sherwood number (Sh) increases with the increase of Schmidt number (Sc) and chemical reaction parameter (Kc).

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Appendix

$$k_1 = \frac{1}{2}\left\{Sc + \sqrt{Sc^2 + 4ScKc}\right\}$$

$$k_2 = \frac{1}{2}\left\{\Pr + \sqrt{\Pr^2 + 4F}\right\}$$

$$k_3 = -\frac{Gm}{k_1^{\,2} - k_1 - N}$$

$$k_4 = -\frac{Gr}{k_2^{\,2} - k_2 - N}$$

$$k_5 = -k_3 - k_4$$

$$k_6 = \frac{1}{2}\left\{1 + \sqrt{1 + 4N}\right\}$$

$$k_7 = k_1^{\,2} k_3^{\,2}$$

$$k_8 = k_2^{\,2} k_4^{\,2}$$

$$k_9 = k_5^{\,2} k_6^{\,2}$$

$$k_{10} = 2k_1 k_2 k_3 k_4$$

$$k_{11} = k_1 + k_2$$

$$k_{12} = 2k_1 k_3 k_5 k_6$$

$$k_{13} = k_1 + k_6$$

$$k_{14} = 2k_2 k_4 k_5 k_6$$

$$k_{15} = k_2 + k_6$$

$$k_{16} = \frac{-\Pr k_7}{4k_1^{\,2} - 2\Pr k_1 - F}$$

$$k_{17} = \frac{-\Pr k_8}{4k_2^{\,2} - 2\Pr k_2 - F}$$

$$k_{18} = \frac{-\Pr k_9}{4k_6^{\,2} - 2\Pr k_6 - F}$$

$$k_{19} = \frac{-\Pr k_{10}}{k_{11}^{\,2} - \Pr k_{11} - F}$$

$$k_{20} = \frac{-\Pr k_{12}}{k_{13}^{\,2} - \Pr k_{13} - F}$$

$$k_{21} = \frac{-\Pr k_{14}}{k_{15}^{\,2} - \Pr k_{15} - F}$$

$$k_{22} = -k_{16} - k_{17} - k_{18} - k_{19} - k_{20} - k_{21}$$

$$k_{23} = \frac{-G\mathrm{r}\, k_{16}}{4k_1^{\,2} - 2k_1 - N}$$

$$k_{24} = \frac{-G\mathrm{r}\, k_{17}}{4k_2^{\,2} - 2k_2 - N}$$

$$k_{25} = \frac{-G\mathrm{r}\, k_{18}}{4k_6^{\,2} - 2k_6 - N}$$

$$k_{26} = \frac{-G\mathrm{r}\, k_{19}}{k_{11}^{\,2} - k_{11} - N}$$

$$k_{27} = \frac{-G\mathrm{r}\, k_{20}}{k_{13}^{\,2} - k_{13} - N}$$

$$k_{28} = \frac{-G\mathrm{r}\, k_{21}}{k_{15}^{\,2} - k_{15} - N}$$

$$k_{29} = \frac{-G\mathrm{r}\, k_{22}}{k_2^{\,2} - k_2 - N}$$

$$k_{30} = -k_{23} - k_{24} - k_{25} - k_{26} - k_{27} - k_{28} - k_{29}$$